

Math 20 Homework # 9

Due May 29, 2013 (Note unusual day of the week)

Do the following problems from the book: 12.1.1, 12.1.2, 12.1.7, 12.1.8, 12.1.10, 12.2.3, 12.2.4, 12.2.6, 12.2.11.

Also, solve the following problems:

1. Let c_n be the n^{th} Catalan number:

$$c_n = \frac{1}{n+1} \binom{2n}{n}.$$

Show that the generating function for the Catalan numbers is

$$C(x) := \sum_{n=0}^{\infty} c_n x^n = \frac{1 - \sqrt{1 - 4x}}{2x}.$$

(Hint: This is easiest if you use the result of Problem 8 on the midterm.)

2. Let $\{S_n\}$ be the random walk in \mathbb{R}^2 , in which we start at the origin, and at each second, we take a step up, down, right, or left, each with probability $1/4$. Let $|(x, y)| = \sqrt{x^2 + y^2}$. Compute $\mathbb{E}(|S_n|^2)$. (Hint: What is $\mathbb{E}(|S_n|^2 - |S_{n-1}|^2)$?)