MATH 20, SPRING 2011
HOMEWORK #2

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This assignment will be due on Wednesday, April 13 at 12:30 p.m. in the box outside 105 Kemeny. Look for the boxes labeled "Math 20, Spring 2011" and put your assignment in the left ("IN") box.

Remember to show your work. A correct answer with no work shown will receive minimal credit. Your solutions should be detailed enough that any of your classmates could understand them simply by reading them.

(1) In a digital computer, a *bit* is one of the integers 0 and 1, a *byte* is a sequence of eight bits, and a *nibble* is a sequence of four bits (I'm not making this up). How many possible bytes and nibbles are there?

(2) How many permutations of the set $\{a, b, c, d, e\}$ start with a and end with c?

(3) If there are twelve students in a class, what is the probability that their birthdays all fall in different months? (Assume that each month is equally likely as a birth month.)

(4) A box contains 30 balls numbered 1 through 30. Suppose that five balls are drawn at random, one at a time, with replacement. What is the probability that the numbers on exactly two of them are prime?

(5) (Section 3.2, #20) A six-card hand is dealt from an ordinary deck of cards. Find the probability that
 (a) all six cards are hearts;
 (b) there are three aces, two kings, and one queen; and
 (c) there are three cards of one suit and three of another suit.

(6) Show that the probability of getting exactly n heads in $2n$ tosses of a fair coin is given by the product of the odd numbers up to $2n - 1$ divided by the product of the even numbers up to $2n$.

(7) Find four sequences $\langle a_n \rangle$, $\langle b_n \rangle$, $\langle c_n \rangle$, and $\langle d_n \rangle$ such that $\langle a_n \rangle \sim \langle b_n \rangle$ and $\langle c_n \rangle \sim \langle d_n \rangle$ but $\langle a_n + c_n \rangle \not\sim \langle b_n + d_n \rangle$ and show that these three relationships hold between them.

Suggested problems: Section 3.1: 5-7, 12-16, 22-23; Section 3.2: 1-3, 6-9, 17, 19