**Math 1: Calculus with Algebra**
**Midterm 1**
**Thursday, October 8**

**Name:** Answer Key

Circle your section number: 1–Freund 2–DeFord

**Please read the following instructions before starting the exam:**

- This exam is closed book, with no calculators, notes, or books allowed. You may not give or receive any help during the exam, though you may ask instructors for clarification if necessary.
- Be sure to **show all work** whenever possible. Even if your final answer is incorrect, we can assign an appropriate amount of partial credit if we can see how you arrived at your answer.
- Please circle or otherwise indicate your final answer if possible.
- The test has a total of 12 questions, worth a total of 120 points. Point values are indicated for each question.
- You will have two hours from the start of the exam to complete it.
- Good luck!

**HONOR STATEMENT:** I have neither given nor received help on this exam, and I attest that all the answers are my own work.

Signature: _____

This page for grading purposes only.

| Problem | Points | Scores |
|---|---|---|
| 1 | 12 | |
| 2 | 12 | |
| 3 | 12 | |
| 4 | 12 | |
| 5 | 12 | |
| 6 | 12 | |
| 7 | 12 | |
| 8 | 12 | |
| 9 | 12 | |
| 10 | 12 | |
| Total | 120 | |

1. **(12 points)** Throughout this problem, let $f(x) = 2x^2 - x + 3$.

**(a)(5 pts)** Find the average rate of change of $f$ over the interval $[-1, 3]$.

$$\frac{f(b)-f(a)}{b-a} = \frac{18-6}{3-(-1)} = \frac{12}{4} = \boxed{3}$$

**(b)(5 pts)** Find the average rate of change of $f$ over the interval $[2, 5]$.

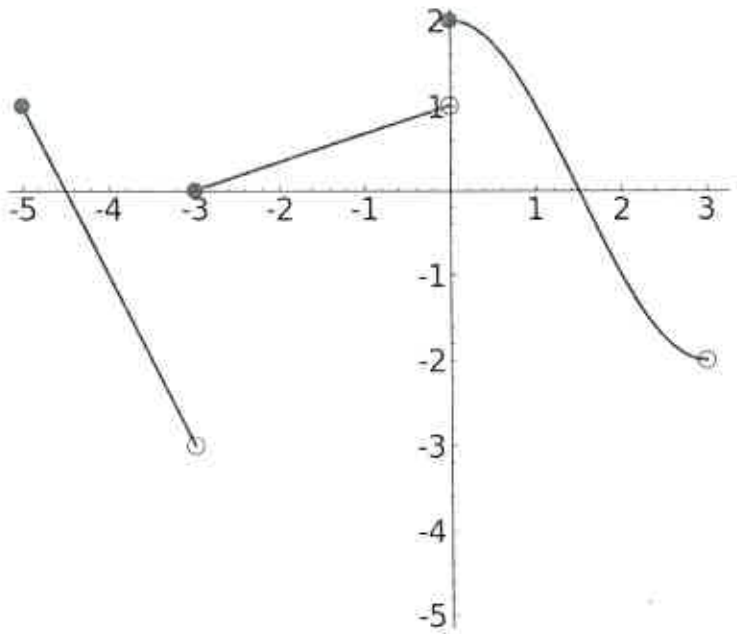
$$\frac{f(b)-f(a)}{b-a} = \frac{48-9}{5-2} = \frac{39}{3} = \boxed{13}$$

**(c)(2 pts)** What is the geometric interpretation of the average rate of change over an interval?

The average rate of change is the slope of the secant line from $(a, f(a))$ to $(b, f(b))$.

2. **(12 points)** *Function Adjectives.*

**(a)(6 pts)** Consider the following graph of $f(x)$ and fill in the information below:




I) The domain of $f$ is: $[-5, 3)$

II) The range of $f$ is: $(-3, 2]$

III) $f$ is increasing on: $[-3, 0)$

IV) $f$ is decreasing on: $[-5, -3) \cup [0, 3)$

**(b)(6 pts)** Consider the functions $g(x) = 2x^2 - \sqrt{x} + 1$ and $h(x) = x^2 - 3x + 2$.

I) What is the domain of $g$?

$[0, \infty)$

II) What is the domain of $\left(\frac{g}{h}\right)$?

Need to rule out zeroes of $h$ as well

$x^2 - 3x + 2 = (x-1)(x-2)$

$[0, 1) \cup (1, 2) \cup (2, \infty)$

III) Write the formula for the function $(h - g)(x)$.

$(x^2 - 3x + 2) - (2x^2 - \sqrt{x} + 1)$

$-x^2 - 3x + \sqrt{x} + 1$

3. **(12 pts)** *Function Composition.*

**(a)(4 pts)** Let $f(x) = \dfrac{2}{x-1}$ and $g(x) = \dfrac{2+x}{x}$. Find and simplify $(f \circ g)(x)$.

$$(f \circ g)(x) = \frac{2}{\frac{2+x}{x} - 1} = \frac{2}{\frac{2+x}{x} - \frac{x}{x}} = \frac{2}{\frac{2}{x}} = x$$

**(b)(4 pts)** Write $h(x) = (x+2)^2$ as a composition of two functions. Show that your decomposition works.

$$f(x) = x^2$$
$$g(x) = x+2$$
$$(f \circ g)(x) = f(x+2) = (x+2)^2$$

**(c)(4 pts)** Write $g(x) = \cos(\ln(x)) + 5\ln(x)$ as a composition of two functions. Show that your decomposition works.

$$f(x) = \cos(x) + 5x$$
$$h(x) = \ln(x)$$
$$(f \circ h)(x) = f(\ln(x)) = \cos(\ln(x)) + 5\ln(x)$$

4. **(12 pts)** *Graph Transformations.*

**(a)(6 pts)** Write the transformations, in order, that are needed to obtain the graph of $g(x) = -\sqrt{x} + 2$ from the graph of $f(x) = \sqrt{x}$.

1) Reflect across x-axis

2) shift up by 2

**(b)(6 pts)** Write the algebraic expression that will transform the graph of the function $h(x) = e^x$ by compressing horizontally by a factor of 2, reflecting across the $x$–axis, and *then* shifting the graph down by $\frac{1}{3}$.

$-e^{2x} - \frac{1}{3}$

5. **(12 points)** *Sequences.*

**(a)(6 pts)** Consider the sequence $\{2n+5\}_{n=0}^{\infty}$.

I) Write out the first 5 terms of this sequence.

5, 7, 9, 11, 13

II) Determine whether the sequence is bounded. Explain your conclusion (1-2 sentences). No. For any bound $M$ the sequence element $a_n > M$.

**(b)(6 pts)** Consider the sequence $\{-3, 6, -9, 12, -15, \ldots\}$.

I) Find a closed form for this sequence.

$\{(-1)^n 3n\}_{n=1}^{\infty}$

II) Determine whether this sequence is increasing, decreasing, or neither.

It is alternating.

6. **(12 pts)** *Inverse Functions.*

**(a)(3 pts)** Find the inverse of the function $f(x) = \dfrac{4}{2x+1}$.

$x = \dfrac{4}{2y+1} \rightarrow 2y+1 = \dfrac{4}{x} \rightarrow 2y = \dfrac{4}{x} - 1$

$y = \dfrac{2}{x} - \dfrac{1}{2}$

**(b)(3 pts)** Compute the domain and range of $f^{-1}(x)$.

Domain $(-\infty, 0) \cup (0, \infty)$

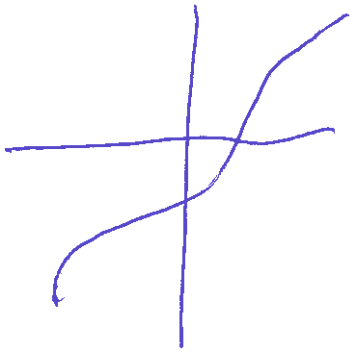
Range $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$

**(c)(3 pts)** Is the function $h(x) = \cos(x)$ one–to–one? If so, explain why. If not find points $a$ and $b$ so that $h(a) = h(b)$. NO.

$\cos(0) = \cos(2\pi)$

**(d)(3 pts)** Is the function $j(x) = 3x^3 - 4$ one–to–one? If so, explain why. If not find points $a$ and $b$ so that $j(a) = j(b)$.

Yes, This Passes the horizontal line test.



7. **(12 pts)** *Simplifying Expressions.*

**(a)(4 pts)** Simplify the expression: $\left(\frac{x^{\frac{1}{2}}z^4y}{y^2}\right)^{-2}$

$$\frac{y^2}{xz^8}$$

**(b)(4 pts)** Solve $3e^{4x+1} = 24$ for $x$.

$$e^{4x+1} = 8$$

$$4x+1 = \ln(8)$$

$$x = \frac{\ln(8)-1}{4}$$

**(c)(4 pts)** Solve $\log_2((x+2)^3) = -6$ for $x$.

$$3\log_2(x+2) = -6$$

$$\log_2(x+2) = -2$$

$$x+2 = 2^{-2}$$

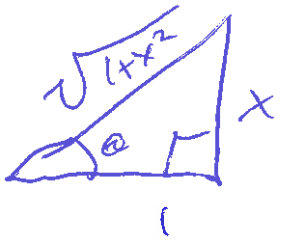
$$x = \frac{1}{4} - 2$$

8. **(12 pts)** *Trigonometry.*

**(a)(6 pts)** Write an algebraic expression for $\cos(\arctan(x))$

$\theta = \arctan(x)$

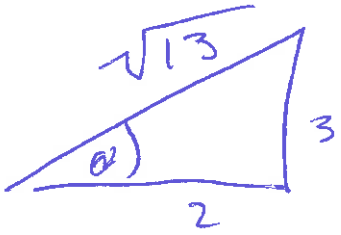
$\tan(\theta) = \frac{x}{1}$




$\cos(\theta) = \frac{1}{\sqrt{1+x^2}}$

**(b)(6 pts)** If $\cot(\theta) = \frac{2}{3}$, find the other 5 trigonometric ratios for $\theta$.

$\cot\theta = \frac{adj}{opp}$




$\sqrt{2^2+3^2} = \sqrt{13}$

$\sin(\theta) = \frac{3}{\sqrt{13}}$ $\quad \csc(\theta) = \frac{\sqrt{13}}{3}$

$\cos(\theta) = \frac{2}{\sqrt{13}}$ $\quad \sec(\theta) = \frac{\sqrt{13}}{2}$

$\tan(\theta) = \frac{3}{2}$ $\quad \cot(\theta) = \frac{2}{3}$

9. **(12 points)** *Multiple Choice.* Please circle your answers.

**(a)(2 pts)** What is the value of $\log_2(\frac{1}{8})$?

I) $\frac{1}{3}$

II) $-3$

III) $-\frac{1}{3}$

IV) 3

**(b)(2 pts)** If $f(x) = x^2 - 2x + 1$, which function represents $f(3x)$?

I) $3x^2 - 6x + 1$

II) $3x^2 - 6x + 3$

III) $9x^2 - 6x + 1$

IV) $9x^2 - 6x + 3$

$f(3x) = (3x)^2 - 2(3x) + 1$

$= 9x^2 - 6x + 1$

**(c)(2 pts)** What is the domain of $\log_4(x)$?

I) $(0, \infty)$

II) $(-\infty, \infty)$

III) $[0, \infty)$

IV) $(-\infty, 0) \cup (0, \infty)$

**(d)(2 pts)** What is $\sin(\arccos(1))$?

I) $\frac{1}{\sqrt{2}}$

II) $-1$

III) 1

IV) 0

**(e)(2 pts)** What is the $x$–intercept of the function $x^2 + 4x + 4$?

I) 2

II) $-2$

III) 0

IV) 4

$x^2+4x+4 = (x+2)^2$

**(f)(2 pts)** What is the $y$–intercept of the function below?

$$e^{12x} - (\ln(x+1) + \tan(x))$$

I) 2

II) 0

III) $-1$

IV) 1

$e^0 - (\ln(1) + \tan(0))$

$1 - (0+0)$

$1$

10. **(12 points)** *True or False.* If the statement is true, explain why. If the answer is false, explain why or give an example that disproves the statement.

**(a)(3 pts)** F If $f$ is a function then $f(2x-1) = 2f(x-1)$.

$f(x) = x^2$ $\quad f(2x-1) = (2x-1)^2$

$2f(x-1) = 2(x-1)^2$

**(b)(3 pts)** F The function $h(x) = x^2 + 3x + 1$ is an even function.

$h(1) = 5$ $\quad h(-1) = -1$

**(c)(3 pts)** F For any two functions $f$ and $g$, $f \circ g = g \circ f$.

$f(x) = x^2$

$g(x) = x+1$

$(f \circ g)(x) = (x+1)^2$

$(g \circ f)(x) = x^2 + 1$

**(d)(3 pts)** T If $f$ is an increasing function, then $f(x+1) > f(x)$ for all $x$.