Math 1: Calculus with Algebra

Sample Exam Questions

Problem 1: Calculate the following limits.

$$\lim_{x\to 0} \cos(\ln(1+x^2))$$

$$\lim_{x\to -3} \frac{\sqrt{x^2+16}-5}{x+3}$$

$$\lim_{x\to -2} \frac{2-|x|}{2+x}$$

$$\lim_{x\to \infty} \frac{3x}{2x^2-9x+1}$$

$$\lim_{x\to 0} \frac{\cos(x)\sin(x)}{x+1}$$

$$\lim_{x\to \pi} \frac{\sin(x)}{x}$$

$$\lim_{x\to \infty} \frac{2-3x^2}{5x^2+x}$$

$$\lim_{x\to 0} \frac{8x^3-7x+1}{-12x^3+2x^2-4}$$

$$\lim_{x \to 0} \frac{x^3}{\tan^3(2x)}$$

$$\lim_{x \to -3} \frac{x^2 - 9}{2x^2 + 7x + 3}$$

$$\lim_{x \to \infty} \left(\sqrt{x^2 + 3x} - \sqrt{x^2 + 8x}\right)$$

$$\lim_{x \to \infty} \frac{\sqrt{9x^6 - 11x^4 - 14}}{8x^3 + 2}$$

$$\lim_{x \to 0} \frac{1 - \cos(x)}{x^2 + x}$$

$$\lim_{x \to \infty} e^{-2x} \sin(x)$$

$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{x^2 + x}\right)$$

$$\lim_{x \to 0} \frac{e^{4x} - 1 - 4x + x^3}{3x^2}$$

Problem 2: Assume that y is a function of x. Calculate $\dfrac{dy}{dx}$.

$y = \ln(18)$

$y = \sin(e^x)$

$2x^4 + xy^3 - y^6 = x^2$

$y = \dfrac{\sin(x)}{x}$

$y = \sec^2(-x)\sqrt{14x^2 - x}$

$y = \sqrt{3x^2}^3$

$y = e^{\frac{3}{x}}$

$y = e^{32}$

$y^x - \ln(xy) = 0$

$\cos(x^2) + \cos(y^2) = \dfrac{x}{y}$

$$y = \csc(4x - 9) + \log_8(x^3)$$

$$y = x^5 \tan(11x)$$

$$y = \ln(x)^{\ln(x)}$$

$$x^3 + y^3 = 11$$

$$y = \sin^{-1}\left(e^{\sqrt{\tan(3x)}}\right)$$

$$y = \sqrt{3x^2}$$

$$y = \sqrt{x + \sqrt{x + \sqrt{x}}}$$

$$y = \frac{e^{2x-1}}{\sec^{-1}(-x^2)}$$

$$y = x^2 + 3x - 11.1$$

$$y = \arctan(\arccos(x))$$

Problem 3: A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 424 ft/s.

(a) At what rate is his distance from second base decreasing when he is halfway to first base?

(b) At what rate is his distance from third base increasing at the same moment?

Problem 4: The altitude of a triangle is increasing at a rate of 1cm/min while the area of the triangle is increasing at a rate of 2cm^2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100cm^2?

Problem 5: Use a linear approximation to estimate $\sqrt{99.8}$.

Problem 6: Find the critical numbers of the function $f(x) = \dfrac{x-1}{x^2+4}$.

Problem 7: Find the absolute maximum and minimum values of $f(x) = x\sqrt{4-x^2}$ on $[-1, 2]$.

Problem 8: Let $f(x) = \sqrt{x^2+1} - x$.

(a) Find any vertical and horizontal asymptotes.

(b) Find any roots of $f(x)$.

(c) Find the intervals of increase or decrease along with any local maximum and minimum values.

(d) Find the intervals of concavity and inflection points.

(e) Use the above information to sketch a graph of $f(x)$.

Problem 9: Sketch the graph of a function which has domain $[-1, 2) \cup (2, \infty)$, range $[-2, 8]$, a horizontal asymptote at $y = 5$, differentiable everywhere except $x = 2, x = 3$ and $x = 4$, discontinuous at $x = 4$, and achieves its maximum at $x = 0$ and achieves its minimum at $x = -1$.

Problem 10: Find the dimensions of a rectangle with area 1000m^2 whose perimeter is as small as possible.

Problem 11: Find the point on the hyperbola $xy = 8$ that is closest to the point $(3, 0)$.

Problem 12: Show that of all the isosceles triangles with a given perimeter, the one with the greatest area is the equilateral.

Problem 13: Use Newton's method to approximate $\sqrt[3]{3}$ using at least three iterations.

Problem 14: Derive the formula for the derivative of some function. Some possibilities include $x^n, a^x, \log_a(x), \sin^{-1}(x), \cos^{-1}(x)$, and the constant function.

Good things to know: Definitions, statements of major theorems covered in class for true-false and short answer.