Math 1: Calculus with Algebra

Sample Exam Questions

Problem 1: Calculate the following limits.

$$\lim_{x\to 3} \frac{x^2 - x - 6}{x - 3}$$

$$\lim_{x\to -4} \frac{\frac{1}{4} + \frac{1}{x}}{4 + x}$$

$$\lim_{x\to 0} \frac{\sin^2(4x)}{x^2}$$

$$\lim_{h\to 0} \frac{(1+h)^5 - 1^5}{h}$$

$$\lim_{x\to 1} x 2^{x^5 - x^3}$$

$$\lim_{x\to \infty} \frac{\sqrt{x^2 - 9}}{2x - 6}$$

$$\lim_{x\to 0} \frac{\sin(\sin(x))}{\sin(x)}$$

$$\lim_{x\to \pi} \ln(\cos(x) + 2)$$

$$\lim_{h\to 0}\frac{\cos(h)-1}{h}$$

$$\lim_{x\to \frac{1}{2}^-}|2x-1|$$

$$\lim_{x\to \frac{1}{2}^+}|2x-1|$$

$$\lim_{x\to\infty}\sqrt{9x^2+x}-3x$$

$$\lim_{x\to 0}\frac{\sin(x)}{x}$$

$$\lim_{x\to\infty}\sqrt{x^3+11x^2-1}$$

$$\lim_{x\to-\infty}\arctan(x)$$

$$\lim_{x\to\infty}\frac{3+e^x}{-11e^x-4}$$

$$\lim_{x\to 1}\frac{\sin(x-1)}{x^2+x-2}$$

Problem 2: The function $[x]$ is defined to be the largest integer that is less than or equal to x, so for example $[3.2]=3$. Graph $f(x)=[x]$. Where is f continuous?

Problem 3: Find the values of a and b that makes the function

$$f(x)=\begin{cases}x^2-3 & \text{if } x<-1\\ ax+b & \text{if } -1\le x\le 1\\ -x^2+4 & \text{if } 1<x\end{cases}$$

continuous everywhere.

Problem 4: Find the equation of the line tangent to the curve at the given point.

a) $y = 4x + 3$ at $x = 2$

b) $y = -4\sin(x) + x^3\cos(x)$ at $x = \dfrac{\pi}{2}$

c) $y = \cos(x)(\tan^2(x)\cos(x) + \cos(x))$ at $x = \dfrac{\pi}{6}$

Problem 5: Compute the following derivatives.

$\dfrac{d}{dx}(\cos(x) - 3x)$

$(x^2e^x)'$

$\dfrac{d}{dx}\left(\dfrac{-3\sec(x) + \sqrt[4]{x}}{-\tan(x)}\right)$

$\left(\dfrac{8\sqrt[3]{x} + \sqrt{7x} - 4x}{x^{3/2}}\right)'$

$(x^3e^x\csc(x))'$

$(e^{2x})'$

Problem 6: Is $f(x) = \sqrt[5]{x}$ differentiable at zero?

Problem 7: Exercises 4-11 from Section 2.8, page 162.

Problem 8: Find the derivative of $f(x) = \dfrac{4x}{x+2}$ using the limit definition of the derivative.

Problem 9: Let $f(x) = \begin{cases} \dfrac{2x^2 - 5x - 3}{x - 3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$. Is f continuous at $x = 3$? Use the definition of continuity to justify your answer.

Problem 10: Find the 1000th derivative of $f(x) = xe^x$.