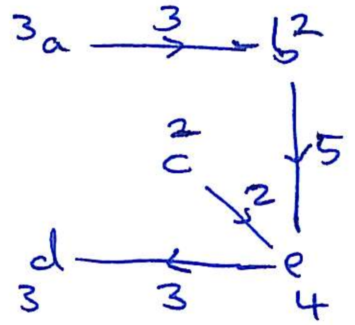
1.



Let e be the root node.

$y_e = 0.$

$y_d - y_e = 13 \Rightarrow y_d = 13$

$y_e - y_c = 3 \Rightarrow y_c = -3$

$y_e - y_b = 7 \Rightarrow y_b = -7.$

$y_b - y_a = -4 \Rightarrow y_a = -3.$

so the dual slacks are

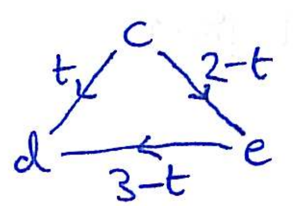
$z_{ac} = -3 + 8 - (-3) = 8$

$z_{bc} = -7 + 6 - (-3) = 2$

$z_{cd} = -3 + 6 - 13 = -10$

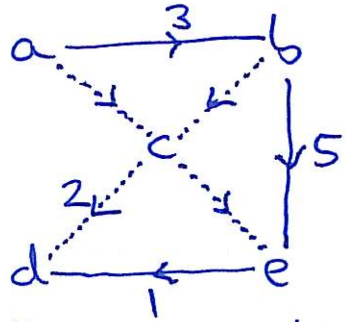
$z_{da} = 13 + 1 - (-3) = 17.$

z_{cd} is infeasible so the current basis is not optimal.
Let (c,d) enter the basis. This forms the cycle



so $t = 2$ and (c,e) leaves the basis.

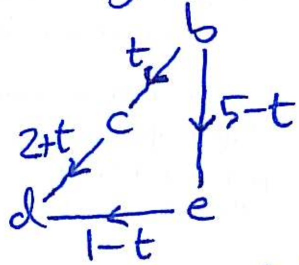
The new basis is



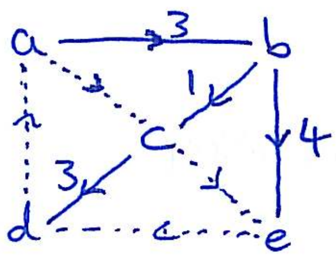
with those arcs whose dual slacks require updating indicated.

$$\tilde{z}_{ac} = 8 + (-10) = -2$$
$$\tilde{z}_{bc} = 2 + (-10) = -8$$
$$\tilde{z}_{ce} = 0 - (-10) = 10$$

For the next iteration, choose arc (b,c) to enter the basis.
This forms the cycle



so $t=1$ and arc (e,d) leaves the basis.
The new solution is



The new dual slacks are

$$\tilde{z}_{ac} = -2 - (-8) = 6$$
$$\tilde{z}_{ce} = 10 + (-8) = 2$$

$\tilde{z}_{da} = 17 + (-8) = 9$

$\tilde{z}_{ed} = 0 - (-8) = 8$

So the current solution is dual and primal feasible, and hence optimal.

The optimal value is

$$3 \cdot (-4) + 1 \cdot 6 + 4 \cdot 7 + 3 \cdot 6 = 40.$$

2. (a.) F. There are only finitely many basic optimal solutions but as ~~long~~ as there are 2 optimal solutions there are infinitely many because any point on the edge connecting these two solutions will also be optimal.

(b.) F. For example, maximise $-x_1$ subject to $x_1 \leq 0$.

(c.) F. The example on p.114 shows that if c_1 is reduced by more than ½ the solution becomes nonoptimal.

(d.) F. We need the y_i's at the outset in order to compute the dual slack variables. Thereafter we do not need them.

(e.) F. We may include a new demand node to represent the surplus, with an arc running to it from each supply node with unit cost of 0 to represent leaving alone any material which flows along that arc in the solution.