Homework: Week 5

A Let $\vec{F}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} e^{y^2} \\ 2xy \end{pmatrix}$ and C be the curve that runs in a straight line from $(0,0)$ to $(0,2)$ and then clockwise along a circular arc to $(2,0)$.

Find the flux of $\vec{F}$ across C.

Hint: computing directly is very, very difficult - try to find a way that involves computing with easier integrals.

B Is the vector field $\vec{F} = \begin{pmatrix} xy^2\sqrt{1000+xy} \\ x^2y\sqrt{1000+xy} \end{pmatrix}$ conservative on the disc of radius 5 around the origin? Justify your answer.

Find $\int_C \vec{F} \cdot d\vec{r}$ where C is the portion of the ellipse $x^2 + 4y^2 = 1$ with $x \geq 0$ and $y \geq 0$, oriented counter-clockwise.

C If r is the radial function $r = \sqrt{x^2+y^2}$ show that

$$\operatorname{curl}\begin{pmatrix} -yf(r) \\ xf(r) \end{pmatrix} \cdot \vec{k} = 2f(r) + rf'(r).$$

Use this to evaluate

$$\iint_D 2\sqrt{1+(x^2+y^2)^{3/2}} + \frac{3}{2}\frac{(x^2+y^2)^{3/2}}{\sqrt{1+(x^2+y^2)^{3/2}}}\, dA$$

where D is the unit circle centered at the origin.