Math 14
Winter 2009
Monday, January 26

Special Homework IV

**Assignment:** Prove Theorem 2 of Section 7.2 of the textbook.

The textbook says, "proved by the same method as Theorem 1." You may follow the proof of Theorem 1 (using your own words, with appropriate citation), but notice that the two theorems are not identical: Line integrals change sign when the orientation is reversed, while path integrals do not. Be sure that you understand why this is, and that your proof is clear about this point.

The following challenge problem is strictly optional. That is, you get no additional credit for doing this problem.

Challenge Problem: Suppose that $\vec{f}(t)$ with domain $[a, b]$ and $\vec{g}(u)$ with domain $[c, d]$ are two different parametrizations of the same oriented curve $\gamma$, and that $\vec{f}$ and $\vec{g}$ are one-to-one functions with continuous nonzero derivative. In particular, this means that for each $t$ in $[a, b]$ there is exactly one $u$ in $[c, d]$ with $\vec{g}(u) = \vec{f}(t)$. Let $h : [a, b] \to [c, d]$ send $t$ to the unique $u$ such that $\vec{g}(u) = \vec{f}(t)$. Show that $h$ is a one-to-one differentiable function. In other words, if we consider $\vec{g}$ to be the original parametrization of $\gamma$ (the $\vec{c}$ of the definition on page 435 of the textbook), then $\vec{f}$ is a reparametrization of $\gamma$ (the $\vec{p}$ of the definition), as $\vec{f}(t) = \vec{g}(h(t))$.

If this is too hard, feel free to make special assumptions. For example, if you assume that no two points of $\gamma$ have the same $x$-component, then the $x$-components of $\vec{f}$ and $\vec{g}$ are invertible functions from $\mathbb{R}$ to $\mathbb{R}$ to which the inverse function theorem applies.