Math 13 Winter 2015 Your name: _____

Instructor (please circle): □ Pierre Clare □ Mingzhong Cai □ Erik van Erp

Problem set 4, due Wed Feb 4

Please show your work. No credit is given for solutions without justification.

(1) Find the volume of the solid that lies below the cone $z = \sqrt{x^2+y^2}$ within the sphere $x^2+y^2+z^2 = 4$, and above the plane $z = 0$.

(2) Find the volume of the solid that lies within the sphere $x^2+y^2+z^2 = 4$ as well as inside the cylinder $x^2+y^2 = 1$.

(3) Find the area of the region R bounded by the hyperbola $xy = 1$ and the lines $y = x$ and $x = 2$. Use the change of variables $u = xy$, $v = x^2$.

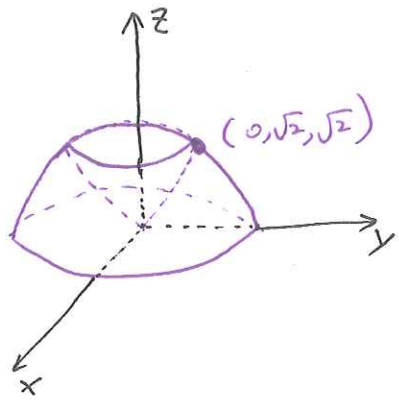
① $V(E) = \iiint_E 1\, dV$

$= \int_{\pi/4}^{\pi/2} \int_0^{2\pi} \int_0^2 r^2 \sin\varphi \, dr\, d\theta\, d\varphi$

$= \frac{r^3}{3}\Big|_{r=0}^{r=2} \cdot \theta\Big|_{\theta=0}^{\theta=2\pi} \cdot -\cos\varphi\Big|_{\varphi=\pi/4}^{\varphi=\pi/2}$

$= \frac{8}{3} \cdot 2\pi \cdot \frac{\sqrt{2}}{2}$

$= \frac{8\sqrt{2}}{3}\pi$



② $V(E) = \iiint_E dV$

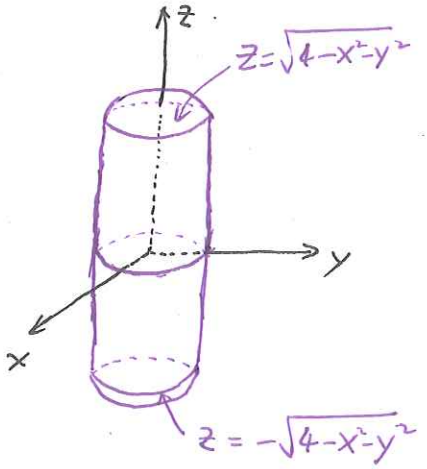
$= \int_0^{2\pi} \int_0^1 \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\, dz\, dr\, d\theta$

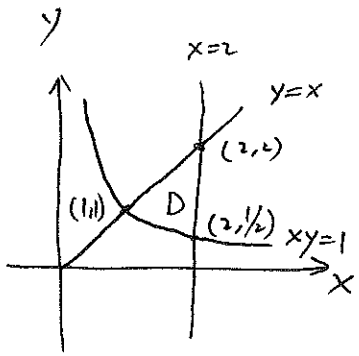
$= \int_0^{2\pi} \int_0^1 2r\sqrt{4-r^2}\, dr\, d\theta$

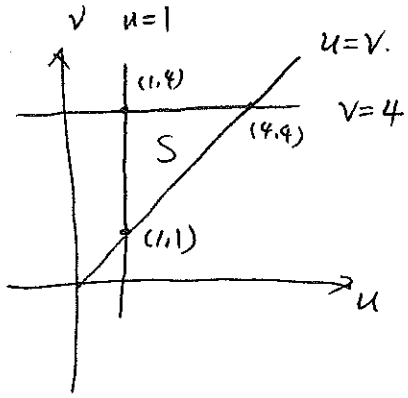
$= 2\pi \cdot -\frac{2}{3}(4-r^2)^{3/2}\Big|_{r=0}^{r=1}$

$= 2\pi\left(-\frac{2}{3}\cdot 3^{3/2} + \frac{2}{3}4^{3/2}\right)$ 1

$= 2\pi\left(\frac{16}{3} - 2\sqrt{3}\right)$







(3) $u = xy$. $v = x^2$.

so $x = \sqrt{v}$, $y = \dfrac{u}{\sqrt{v}}$.

on the uv-plane,

$x = 2$ becomes $\sqrt{v} = 2$, or $v = 4$.

$xy = 1$ becomes $u = 1$

$x = y$ becomes $\sqrt{v} = \dfrac{u}{\sqrt{v}}$, or $v = u$.

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 0, & \frac{1}{2\sqrt{v}} \\ \frac{1}{\sqrt{v}}, & \frac{-u}{2v\sqrt{v}} \end{vmatrix} = -\frac{1}{2v}$$

$$A(D) = \iint_D 1 \, dA = \int_1^4 \int_1^v \frac{1}{2v} \, du \, dv$$

$$= \int_1^4 \frac{1}{2v}(v-1) \, dv$$

$$= \int_1^4 \left(\frac{1}{2} - \frac{1}{2v}\right) dv$$

$$= \left(\frac{1}{2}v - \frac{1}{2}\ln|v|\right)\Big|_{v=1}^{v=4}$$

$$= \left(\frac{3}{2} - \frac{1}{2}\ln 4\right)$$

$$= \frac{3}{2} - \ln 2.$$