Math 13 Winter 2015 Your name: ______________________

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Problem set 1, due Wed Jan 14

Please show your work. No credit is given for solutions without justification.

(1) Calculate the average value of the product xy for the values $0 \le x \le 10$, $0 \le y \le 10$.

(2) (a) Approximate the volume of the solid region below the plane $x + y + z = 10$ and above the rectangle $1 \le x \le 4$, $2 \le y \le 3$ in the xy-plane, by evaluating a Riemann Sum with $\Delta x = 1.0$, $\Delta y = 0.5$. Use the *midpoint rule* to select sample points.
(b) Calculate the exact value of this volume.

(3) Let $\mathcal{D}$ be the half circle with $x \ge 0$ and $x^2 + y^2 \le 1$. Evaluate the double integral

$$\iint_{\mathcal{D}} xy^2 \, dA$$

①. $f_{ave} = \frac{1}{A(R)} \cdot \iint_R xy \, dA$

$= \frac{1}{10 \times 10} \cdot \int_0^{10} \int_0^{10} xy \, dy \, dx$

$= \frac{1}{100} \cdot \int_0^{10} x \cdot \left(\frac{1}{2}y^2\right)\Big|_{y=0}^{y=10} dx$

$= \frac{1}{100} \cdot \frac{100}{2} \cdot \int_0^{10} x \, dx$

$= \frac{1}{100} \cdot \frac{100}{2} \cdot \frac{100}{2}$

$= 25.$

② . $z = 10 - x - y$ and midpoints are $(1.5, 2.25)$. $(1.5, 2.75)$

$(2.5, 2.25)$. $(2.5, 2.75)$

$(3.5, 2.25)$. $(3.5, 2.75)$

andso

$$\text{Riemann Sum} = \Delta x \cdot \Delta y \cdot \big((10-1.5-2.25) + (10-1.5-2.75) + (10-2.5-2.25) + (10-2.5-2.75) + (10-3.5-2.25) + (10-3.5-2.75) \big)$$

$$= 1 \times 0.5 \times \big(60 - 1.5\times 2 - 2.5\times 2 - 3.5\times 2 - 3\times(2.25+2.75) \big)$$

$$= 0.5 \times (60 - 3 - 5 - 7 - 3\times 5)$$

$$= 15.$$

$$\text{Volume} = \iint_R (10-x-y)\, dA$$

$$= \int_1^4 \int_2^3 (10-x-y)\, dy\, dx$$

$$= \int_1^4 \int_2^3 10\, dy\, dx - \int_1^4 \int_2^3 x\, dy\, dx - \int_1^4 \int_2^3 y\, dy\, dx$$

$$= 10\cdot(4-1)\cdot(3-2) - \int_1^4 x\cdot(3-2)\, dx - \int_1^4 \tfrac{1}{2} y^2 \Big|_{y=2}^{y=3} dx$$

$$= 30 - \tfrac{1}{2} x^2 \Big|_{x=1}^{x=4} - \tfrac{5}{2}\cdot(4-1)$$

$$= 30 - \tfrac{15}{2} - \tfrac{15}{2}$$

$$= 15.$$

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$$\iint_D xy^2\,dA = \int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}} xy^2\,dx\,dy$$

$$= \int_{-1}^{1} y^2 \, \frac{1}{2}x^2 \Big|_{x=0}^{x=\sqrt{1-y^2}} dy$$

$$= \int_{-1}^{1} \frac{1}{2} y^2(1-y^2)\,dy$$

$$= \frac{1}{2}\left(\int_{-1}^{1} y^2 - y^4\,dy\right)$$

$$= \frac{1}{2}\left(\frac{1}{3}y^3 - \frac{1}{5}y^5\right)\Big|_{y=-1}^{y=1}$$

$$= \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5} - \left(-\frac{1}{3} + \frac{1}{5}\right)\right)$$

$$= \frac{2}{15}.$$