Practice Exam 2

1. A wire is bent in the shape of the curve $y = x^3$, $0 \leq x \leq 1$. Find the total mass of the wire if the density is given by $\rho(x, y) = y$.

2. Let $\mathbf{F}(x, y, z) = \langle 2xyz, x^2z + z + 1, x^2y + y + 1 \rangle$.

(a) Find a function f such that $\mathbf{F} = \nabla(f)$.

(b) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the curve given by $\mathbf{r}(t) = \langle \frac{t}{4}, \sqrt{9 + t^2}, \frac{t}{2} \rangle$, $0 \leq t \leq 4$.

3. Match each vector field with its plot.

(a) ______ $\mathbf{F}(x, y) = \langle \frac{1}{5}, \frac{x}{5} \rangle$

(b) ______ $\mathbf{F}(x, y) = \langle \frac{x}{5}, \frac{1}{5} \rangle$

(c) ______ $\mathbf{F}(x, y) = \nabla(\frac{x^2+y^2}{10})$

(d) ______ $\mathbf{F}(x, y) = \langle \frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}} \rangle$

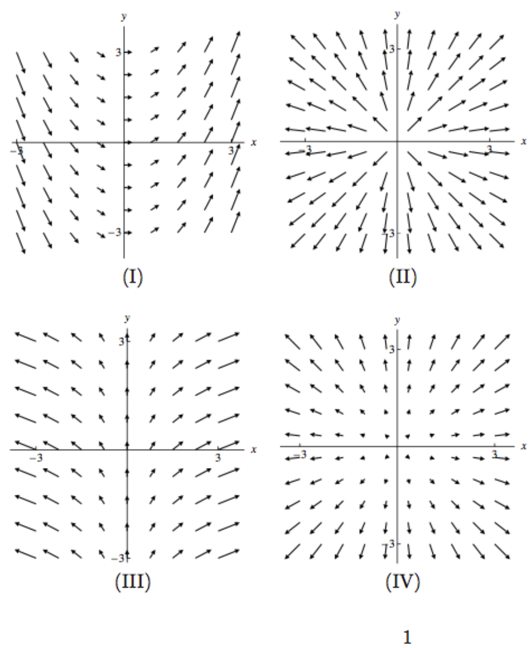

Figure 1:

4. Compute each of the following line integrals by any appropriate method. If you use a method other than direct computation, then include the following:

- Indicate what technique you are using.
- Justify that the technique can be used. (E.g., if you are using techniques applicable only to conservative vector fields, show that the vector field is conservative.)
- Apply the technique to compute the line integral.

(a)

$$\int_C xy\,dx + x^2\,dy$$

where C is the straight line segment from $(0, 2)$ to $(4, 5)$.

(b)

$$\int_C (2xy+1)\,dx \;+\; (x^2+e^{y^2})\,dy$$

where C is given by $\mathbf{r}(t) = (t,\, t^2 - t)$, $0 \le t \le 1$.

(c)

$$\int_C y\,dx \; + \; (2x + e^{y^2})\,dy$$

where C is the circle given by $\mathbf{r}(t) = (1 + 2\cos(t), 2\sin(t))$, $0 \le t \le 2\pi$.

5. Each of the following vector fields satisfies the condition "$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$". (You do not need to verify this fact.) In each case, determine whether the vector field is conservative. Justify your answer.

   (a) $\mathbf{F}(x, y) = \langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \rangle$

(b) (See instructions on previous page.) $\mathbf{F}(x, y) = \langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \rangle$

6. Find the area enclosed by the curve $\mathbf{r}(t) = \langle t^3 - t, (t - \frac{1}{2})^2 \rangle$, $0 \leq t \leq 1$.

7. Let R be the parallelogram with vertices $(0,0)$, $(4,1)$, $(1,2)$ and $(5,3)$. Evaluate

$$\iint_R x dA$$

by first making an appropriate change of variable so that the region of integration is a square in the u,v-plane.