Math 13 Winter 2014

First Practice Exam

1. Let S be the surface
$$xyz + z^3 = 2.$$
Which of the following lines is orthogonal to the tangent plane to S at the point $(1, 1, 1)$?

   (a) $\mathbf{r}(t) = \langle 1 + 2t, 1 + 2t, 1 + 5t \rangle$

   (b) $\mathbf{r}(t) = \langle 1 + t, 1 + t, 1 + 4t \rangle$

   (c) $\mathbf{r}(t) = \langle 1 + 2t, 1 + 2t, 1 + t \rangle$

   (d) $\mathbf{r}(t) = \langle 1 + 2t, 1 + 2t, 1 - 3t \rangle$

   (e) $\mathbf{r}(t) = \langle 1 + t, 1 + t, 1 - 5t \rangle$

   (f) none of the above

2. Suppose that f is a differentiable function from $\mathbf{R}^2$ to $\mathbf{R}$, that $f(1, 2) = 6$ and $\nabla f(1, 2) = \langle 3, 4\rangle$ where ∇f denotes the gradient of f.

(a) Find the direction and rate of maximum increase of f at the point $(1, 2)$.

(b) Find the equation of the tangent plane to the graph of f at the point $(1, 2, 6)$.

(c) (This is a continuation of the problem on the previous page. In particular, we are assuming that $f(1,2) = 6$ and $\nabla f(1,2) = \langle 3, 4 \rangle$.)

Find the directional derivative of f at $(1,2)$ in the direction from $(1,2)$ towards $(2,3)$.

(d) A certain unit vector $\mathbf{u}$ makes an angle of $\frac{\pi}{3}$ with the vector $\nabla f(1,2) = \langle 3, 4 \rangle$. Find the directional derivative of f at $(1,2)$ in the direction $\mathbf{u}$.

3. Let $f(x, y) = ((x + 2y)^2, x^2y^4)$. Compute the matrix $f'(1, 1)$.

4. Let T be the linear transformation with matrix

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 2 & 3 \end{bmatrix}$$

and let S be the linear transformation given by

$$S(x, y) = (x + y, 2x + 3y).$$

(a) Write down the representing matrix for S.

(b) Does the composition $S \circ T$ make sense? If so, write down its representing matrix.

(c) Does the composition $T \circ S$ make sense? If so, write down its representing matrix.

5. A parallelepiped P has one vertex at the point $(1, 0, 1)$, and the three vertices adjacent to this vertex are at $(2, 2, 2)$, $(4, 1, 1)$, and $(2, 2, 3)$. Find the volume of P.

6. Evaluate by any convenient method.

$$\int_{-1}^{1} \int_{|y|}^{1} \cos(x^2)\, dx\, dy.$$

7. Convert to cylindrical coordinates. Do not evaluate.

$$\int_0^1 \int_1^{\sqrt{2-x^2}} \int_0^{x^2+y^2} xy\, dz\, dy\, dx.$$

8. Evaluate

$$\int_{-\sqrt{2}}^{-1}\int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} dy\,dx + \int_{-1}^{1}\int_{x}^{\sqrt{2-x^2}} dy\,dx$$

(Note: You can do this problem without computation if you think geometrically.)

9. Write down an iterated integral that gives the total mass of the tetrahedron with vertices $(1, 0, 0)$, $(2, 0, 0)$, $(1, 0, 1)$ and $(1, 1, 0)$ if the density at each point (x, y, z) is given by xy. You do *not* have to evaluate the integral.

10. Match the following with their descriptions. You might use a description more than once.

_______ $\varphi = \frac{\pi}{2}$ (this is in spherical coordinates)

_______ $z = 5r$ (this is in cylindrical coordinates)

_______ $r = 2\cos(\theta)$ (this is in cylindrical coordinates)

_______ $\rho\cos(\varphi) = 1$ (this is in spherical coordinates)

_______ a level surface of the function $f(x, y, z) = x - y$.

_______ a level surface of the function $f(x, y, z) = \frac{\sqrt{x^2+y^2}}{5z}$.

(a) a cylinder (This means an actual cylinder. In particular, the cross-sections are round.

(b) a cone

(c) a plane

(d) a sphere

(e) a line

(f) a ray

11. Calculate the volume of the solid E which lies inside the sphere $x^2 + y^2 + z^2 = 16$ and outside the cylinder $r = 4\cos\theta$.