Worksheet March 4

Let C_1 be the circle $x^2 + y^2 = 1$, $z = 0$, and let C_2 be the circle $x^2 + y^2 = 1$, $z = 2$. Assume that both circles are oriented counterclockwise when viewed from above. Let

$$\mathbf{F}(x, y, z) = \langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2}, z^3 \rangle.$$

Use two different methods to show that

$$\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_2} \mathbf{F} \cdot d\mathbf{r}.$$

- Method 1: Compute both line integrals and compare.
- Method 2: Let S be the cylinder $x^2 + y^2 = 1$, $0 \leq z \leq 2$, with the "outward' normal. (The surface S does not contain the bottom and top disks.) Note that the boundary of S consists of the two circles above, although you need to determine whether they are oriented correctly. Then apply Stokes Theorem.