Worksheet January 8

(1) Sketch the contour map of the function $f(x, y) = \frac{1}{x-y^2}$.

(2) Let $f(x, y) = 3x^2 + 4y^3$.
   (a) Find $\nabla f(1, 1)$.

   (b) Find the equation of the tangent plane to f at (1,1,7).

   (c) Compute the directional derivative $D_{\mathbf{u}}f(1, 1)$ where $\mathbf{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$.

   (d) Compute the directional derivative of f at (1,1) in the direction from (1,1) towards the origin.

(3) Let $f(x, y, z) = (3x^2 + 4y^3, xyz)$. Write down the derivative matrix $f'(1, 1, 2)$ of f at (1,1,2).

(4) Suppose $f : \mathbf{R}^2 \to \mathbf{R}^2$ is a differentiable function. Write $(u, v) = f(x, y)$. Suppose

$$f'(2, 1) = \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix}$$

Read off the partial derivatives $\frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}$.