Math 13 - Winter 2014
Homework 8
Due Wednesday, 5 Mar. 2014.

Note:

- Except for problems that are stated explicitly, all problems are from Stewart Multivariable Calculus 7th Edition.
- Please show all of your work (writing a list of answers is not sufficient).
- Please indicate the people you worked with.
- Please staple your page together.

1. (3 pts) Evaluate
$$\int\int_S y^2 dS,$$
where S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy-plane.

2. (3 pts) Verify the Divergence Theorem is true for the vector field $\mathbf{F} = x^2\mathbf{i} + xy\mathbf{j} + z\mathbf{k}$, where E is the solid given by paraboloid $z = 4 - x^2 - y^2$ and the xy-plane.

3. Carry out the following steps to evaluate
$$\int\int_S \mathbf{F} \cdot d\mathbf{S},$$
where $\mathbf{F} = (x + \sin z)\mathbf{i} + (x + e^{z^2})\mathbf{j} + (1 + z)\mathbf{k}$ and S is the hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$.

   (a) (3 pts) Use the Divergence Theorem to evaluate the
$$\int\int_{S_1} \mathbf{F} \cdot d\mathbf{S},$$
where S_1 is the boundary of the half ball $x^2 + y^2 + z^2 \leq 1$, $z \geq 0$ with positive orientation.

   (b) (3 pts) Evaluate the surface integral over S_2 (bottom surface of the half ball given in part (a))
$$\int\int_{S_2} \mathbf{F} \cdot d\mathbf{S},$$

(c) (3 pts) By combining the information from part (a) and (b), Find

$$\int\int_S \mathbf{F} \cdot d\mathbf{S}$$