Math 13 - Winter 2014
Homework 4
Due Wednesday, 19 Feb. 2014.

Note:

- Except for problems that are stated explicitly, all problems are from Stewart Multivariable Calculus 7th Edition.
- Please show all of your work (writing a list of answers is not sufficient).
- Please indicate the people you worked with.
- Please staple your page together.

1. (3pts) Find the work done by the force field $\mathbf{F}(x, y) = x^2\mathbf{i} + ye^x\mathbf{j}$ on a particle that moves along the parabola $x = y^2 + 1$ from $(1, 0)$ to $(2, 1)$.

2. (a)(1.5pts) Find a function f such that $\mathbf{F}(x, y, z) = (y^2z + 2xz^2)\mathbf{i} + (2xyz)\mathbf{j} + (xy^2 + 2x^2z)\mathbf{k} = \nabla f$

   (b)(1.5pts) Use part (a) to evaluate
   $$\int_C \mathbf{F} \cdot d\mathbf{r},$$
   where
   $$C : x = \sqrt{t},\ y = t + 1,\ z = t^2,\ 0 \leq t \leq 1.$$

3. Compute the following line integrals. You may use any method that is applicable to the given problem. (E.g., if the vector field is conservative, you may use methods that apply to conservative vector fields.)

   (a) (1.5pts) $\int_C (ye^{xy} + x)dx + (xe^{xy} + y)dy$ where C is the semi-ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$, $y \geq 0$ traced from $(-2, 0)$ to $(2, 0)$.

   (b) (1.5pts) $\int_C e^{x^2}\, dx + y\, dy$ where C is the semi-ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$, $x \geq 0$ traced from $(0, -3)$ to $(0, 3)$.

4. (3pts) Verify Green's Theorem by evaluating the line integral below by (a) using Green's Theorem, and (b) by direct evaluation.

$$\int_C xy\ dx + x^2\ dy,$$

where C is the rectangle (oriented counterclockwise) with vertices $(0,0)$, $(3,0)$, $(3,1)$, and $(0,1)$.

5. (3pts) Use Green's Theorem to evaluate

$$\int_C (1-y^3)dx + (x^3 + e^{y^2})dy,$$

where C is the boundary of the region between the circles $x^2+y^2=4$ and $x^2+y^2=9$.

6. (3pts) A particle starts at the point $(-2,0)$, moves along the x-axis to $(2,0)$, and then along the semicircle $y=\sqrt{4-x^2}$ to the starting point. Use Green's Theorem to find work done on this particle by the force field $\mathbf{F}(x,y) =< x, x^3+3xy^2 >$