Math 13: Written Homework #7.
Due Monday, February 25, 2013.

1. (§16.4, #22) Let D be the region bound by a simple positively oriented closed path C in the xy-plane. Use Green's Theorem to prove that the coordinates $(\bar{x}, \bar{y})$ of the center of mass of D (assuming D is a lamina of constant density) are given by

$$\bar{x} = \frac{1}{2A} \int_C x^2\, dy \quad \text{and} \quad \bar{y} = -\frac{1}{2A} \int_C y^2\, dx,$$

where A is the area of D.

2. (§16.5, #20) Is there a smooth vector field $\mathbf{G}$ on $\mathbf{R}^3$ such that $\boldsymbol{\nabla} \times \mathbf{G} = \langle xyz, -y^2 z, yz^2 \rangle$? Justify your assertions.

3. Suppose that D is a subset of $\mathbf{R}^3$ and that f is a scalar valued function on D while $\mathbf{F}$ is a vector field on D. Assuming both f and the components of $\mathbf{F}$ have continuous partial derivatives, show that

$$\operatorname{div}(f\mathbf{F}) = f \operatorname{div}(\mathbf{F}) + \mathbf{F} \cdot \boldsymbol{\nabla} f.$$

4. (§16.6, #24) Find a parametric representation for the surface which is the part of the sphere $x^2 + y^2 + z^2 = 16$ which lies between the planes $z = -2$ and $z = 2$.

5. (§16.6, #26) Find a parametric representation of the part of the plane $z = x + 3$ which lies inside the cylinder $x^2 + y^2 = 1$.

6. (§16.6, #36) Let $\mathbf{r}(u, v) = \langle \sin u, \cos u \sin v, \sin v \rangle$. Find an equation for the tangent plane to the surface parameterized by $\mathbf{r}$ when $u = \pi/6$ and $v = \pi/6$.