Math 13: Written Homework #3.
Due Monday, January 28, 2013.

1. Find the volume of the solid that lies between the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 2$.

2. (§15.8 #28) Find the mass of the ball B given by $x^2 + y^2 + z^2 \leq a^2$ if the density at any point of the ball is proportional to its distance from the z-axis. (You may do the problem any way you wish, but spherical coordinates give a simpler integral.)

3. (§15.9 #28) Find the average distance of a point in a solid ball of radius a to its center.

4. (§12.4 #48) Suppose that $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are vectors in $\mathbf{R}^3$ such that $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$. Show that $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{c} = \mathbf{c} \times \mathbf{a}$.

5. (§15.10 #18) Evaluate

$$\iint_R (x^2 - xy + y^2)\, dA,$$

where R is the region bounded by the ellipse $x^2 - xy + y^2 = 2$. Use the change of variables $x = \sqrt{2}u - \sqrt{2/3}v$ and $y = \sqrt{2}u + \sqrt{2/3}v$.

6. (§15.10 #14) Let R be the region in the first quadrant bounded by the hyperbolas $y = 1/x$, $y = 4/x$, and the lines $y = x$ and $y = 4x$. Find the equations for the transformation T that maps a rectangular region S of the uv-plane onto R, where the sides of S are parallel to the u- and v-axes.