MATH 13, WINTER 2011
WRITTEN HOMEWORK #8

JOHANNA N.Y. FRANKLIN

This assignment will be due on Wednesday, March 2 at 12:30 p.m. in the boxes outside 105 Kemeny. Look for the column of boxes labeled "Math 13, Winter 2011" and put your assignment in the left ("IN") column corresponding to the first letter of your family name (A-F, G-M, N-S, T-Z).

Remember to show your work. A correct answer with no work shown will receive minimal credit. Your solutions should be detailed enough that any of your classmates could understand them simply by reading them.

(1) (17.7, #28) Find the flux of $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + 5\mathbf{k}$ across S, where S is the boundary of the region enclosed by the cylinder $x^2 + z^2 = 1$ and the planes $y = 0$ and $x + y = 2$. Use the positive orientation for S.

(2) (17.7, #30) Find the flux of $\mathbf{F}(x, y, z) = y\mathbf{i} + (z - y)\mathbf{j} + x\mathbf{k}$ across S, where S is the surface of the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$. Use the positive orientation for S.

(3) (17.7, #36) Find a formula for $\iint_S \mathbf{F} \cdot d\mathbf{S}$ similar to Formula 10 for the case where S is given by $x = k(y, z)$ and $\mathbf{n}$ is the unit normal vector that points forward (that is, toward the viewer when the axes are drawn in the usual way).

(4) (17.7, #38) Find the mass of a thin funnel in the shape of a cone $z = \sqrt{x^2 + y^2}$, $1 \leq z \leq 4$, if its density function is $\rho(x, y, z) = 10 - z$.

(5) (17.7, #46) The temperature at a point in a ball with conductivity K is inversely proportional to the distance from the center of the ball. Find the rate of heat flow across a sphere of radius a with center at the center of the ball.

(6) (17.7, #47) Let $\mathbf{F}$ be an inverse square field, that is, $\mathbf{F}(\mathbf{r}) = \frac{c\mathbf{r}}{|\mathbf{r}|^3}$ for some constant c, where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Show that the flux of F across a sphere S centered at the origin is independent of the radius of S.

(7) (17.9, #4) Verify that the Divergence Theorem is true for the vector field $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ on the unit ball $x^2 + y^2 + z^2 \leq 1$.

Suggested problems: 17.7: 9-12, 22-24, 35, 37, 42; 17.9: 1-3, 5-6