MATH 13, WINTER 2011 WRITTEN HOMEWORK #5

JOHANNA N.Y. FRANKLIN

This assignment will be due on Wednesday, February 9 at 12:30 p.m. in the boxes outside 105 Kemeny. Look for the column of boxes labeled "Math 13, Winter 2011" and put your assignment in the left ("IN") column corresponding to the first letter of your family name (A-F, G-M, N-S, T-Z).

Remember to show your work. A correct answer with no work shown will receive minimal credit. Your solutions should be detailed enough that any of your classmates could understand them simply by reading them.

- (1) (16.9, #10 and #22)
 - (a) Find the image of the disk given by $u^2 + v^2 \leq 1$ under the transformation x = au, y = bv. Then sketch this disk and its image on separate graphs.
 - (b) Evaluate the integral $\iint_R \sin(9x^2 + 4y^2) dA$ where R is the region in the first quadrant bounded by the ellipse $9x^2 + 4y^2 = 1$.
- (2) (16.9, #24) Let f be continuous on [0, 1] and let R be the triangular region with vertices (0, 0), (1, 0), and (0, 1). Show that $\iint_R f(x+y) \, dA = \int_0^1 u f(u) \, du$.
- (3) (17.2, #34) A thin wire has the shape of the first-quadrant part of the circle with center the origin and radius a. If the density function is $\rho(x, y) = kxy$, find the mass and center of mass of the wire.
- (4) (17.2, #46) The base of a circular fence with radius 10 m is given by $x = 10 \cos t$, $y = 10 \sin t$. The height of the fence at position (x, y) is given by the function $h(x, y) = 4 + 0.01(x^2 - y^2)$, so the height varies from 3 m to 5 m. Suppose that 1 L of paint covers 100 m². How much paint will you need to paint both sides of the fence?