MATH 13, WINTER 2011
WRITTEN HOMEWORK #2

JOHANNA N.Y. FRANKLIN

This assignment will be due on Wednesday, January 19 at 12:30 p.m. in the boxes outside 105 Kemeny. Look for the column of boxes labeled "Math 13, Winter 2011" and put your assignment in the left ("IN") column corresponding to the first letter of your family name (A-F, G-M, N-S, T-Z).

Remember to show your work. A correct answer with no work shown will receive minimal credit. Your solutions should be detailed enough that any of your classmates could understand them simply by reading them.

(1) (Chapter 14 Review, #8) Find the length of the curve $\mathbf{r}(t) = \langle 2t^{\frac{3}{2}}, \cos 2t, \sin 2t \rangle$ on the interval $0 \leq t \leq 1$.

(2) (Chapter 14 Review, #18) A particle starts at the origin with initial velocity $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$. Its acceleration is $\mathbf{a}(t) = 6t\mathbf{i} + 12t^2\mathbf{j} - 6t\mathbf{k}$. Find its position function.

(3) (15.6, #55) Show that every plane that is tangent to the cone $x^2 + y^2 = z^2$ passes through the origin.

(4) (16.1, #17 and #18)
   (a) Show that if f is a constant function of two variables (so $f(x, y) = k$ for some constant k) and $R = [a, b] \times [c, d]$, then $\iint_R k \, dA = k(b - a)(d - c)$.
   (b) Use part (a) to show that $0 \leq \iint_R sin(\pi x)cos(\pi y) \, dA \leq \frac{1}{32}$ where $R = [0, \frac{1}{4}] \times [\frac{1}{4}, \frac{1}{2}]$.

(5) For this problem (and in general), remember that an example is not enough. For instance, in part (a), I want you to tell me the value of $\iint_R f(x, y) \, dA$ for all f with the property given, not just some sample f. You can do the calculations for some functions with that property to figure out what the answer must be, but you must explain why your answer is true for *any* function with that property.
   (a) Suppose that for all x and y, $f(x, y) = -f(-x, -y)$. What must $\iint_R f(x, y) \, dA$ be when R is $[-a, a] \times [-a, a]$?
   (b) Suppose that for all x and y, $f(x, y) = f(x, -y)$. If R is $[-a, a] \times [-b, b]$ and S is $[-a, a] \times [0, b]$, what is the relationship between $\iint_R f(x, y) \, dA$ and $\iint_S f(x, y) \, dA$?