Each problem is worth 7.5 points.

1. Which of the following equations represents the plane passing through $(-5, 2, 3)$ and orthogonal to the line $(2 + t, -7t, -11 + 2t)$?

   (a) $-5(x-1) + 2(y+7) + 3(z-2) = 0$

   (b) $x - 7y + 2z = -13$

   (c) $x - 7y + 2z = -3$

   (d) $x + y + z = 1$

2. An equation of the tangent line to the curve $c(t) = (t^2 - 1, \cos(t^2), t^4)$ at the point $(\pi - 1, -1, \pi^2)$ is:

   (a) $(2\sqrt{\pi}t - \pi - 1, -2t, 4\pi\sqrt{\pi}t - 3\pi^2)$

   (b) $(2\sqrt{\pi}t + \pi - 1, -2t, 4\pi\sqrt{\pi}t + \pi^2)$

   (c) $(2\sqrt{\pi}t - \pi - 1, -1, 4\pi\sqrt{\pi}t - 3\pi^2)$

   (d) $(2\sqrt{\pi}t + \pi, -1, 4\pi\sqrt{\pi}t + \pi^2)$

3. Match the following functions with their level curves $f(x, y) = k$, $k = 1, 2, 3, 4, ...$

(1) $f(x,y) = (x^2 + y^2)^{1/2}$	(*i*) unequally spaced concentric circles
(2) $f(x,y) = 4 - 3x + 2y$	(*ii*) unequally spaced lines
(3) $f(x,y) = 2x^2 + 2y^2$	(*iii*) concentric ellipses
(4) $f(x,y) = x^2 + 2y^2 + 1$	(*iv*) equally spaced concentric circles

   (a) 1-(iv), 2-(ii), 3-(i), 4-(iii)

   (b) 1-(i), 2-(ii), 3-(iv), 4-(iii)

   (c) 1-(iii), 2-(ii), 3-(i), 4-(i)

   (d) 1-(ii), 2-(ii), 3-(iv), 4-(iii)

4. An equation of the tangent plane to the surface $x^2 + y^2 + xy\sin z - 3 = 0$ at the point $(1, -2, \frac{\pi}{2})$ is:

   (a) $y + z = 1 + \frac{\pi}{12}$

   (b) $3y + 2z = 6 + \frac{\pi}{2}$

   (c) $y = -2$

   (d) $3y = 2$

5. Consider the functions $f(u, v) = e^{uv}$ and $g(x, y) = (x + y, x - y)$. The derivative matrix of $f \circ g$ at $(x, y) = (1, 1)$ is equal to:

   (a) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

   (d) $\begin{bmatrix} 2 & -2 \end{bmatrix}$

6. The directional derivative of $f(x,y,z) = \frac{1}{x^2+y^2+z^2}$ at the point $(2,3,1)$ in the direction of $\mathbf{v} = 2\ \mathbf{i} + \mathbf{j} - 2\ \mathbf{k}$ is:

(a) $-\frac{5}{21\cdot 14}$

(b) $\frac{5}{21\cdot 14}$

(c) $-\frac{5}{7\cdot 14}$

(d) $\frac{5}{7\cdot 14}$

7. The length of the curve $c(t) = t\ \mathbf{i} + \ln t\ \mathbf{j} + 2\sqrt{2t}\ \mathbf{k}$ for $1 \le t \le 2$ is:

(a) $2 + \ln 2$

(b) $\frac{19}{2} + \frac{\ln^3 8}{2}$

(c) $1 + \ln 2$

(d) 2

8. Which of the following implies the vector field $F = F_1\mathbf{i} + F_2\mathbf{j} + F_3\mathbf{k}$ is not a gradient?

(a) $\frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x}$, $\frac{\partial F_1}{\partial z} = \frac{\partial F_3}{\partial x}$ and $\frac{\partial F_2}{\partial z} = \frac{\partial F_3}{\partial y}$

(b) $\nabla \times F = 0$

(c) $\int_{C_1} F \cdot d\mathbf{s} = \int_{C_2} F \cdot d\mathbf{s}$, where C_1 and C_2 are any two curves with common start point and common end point.

(d) $\int_C F \cdot d\mathbf{s} = 1$, where C is a closed curve.

9. The volume of the solid below the surface $z = (xy)^2 \cos(x^3)$ and above the region R in the xy-plane given by $R = [0, \sqrt[3]{\frac{\pi}{4}}] \times [0,1]$ is equal to:

(a) $\frac{\sqrt{2}}{18}$

(b) $\frac{1}{18}$

(c) $\frac{\sqrt{2}}{6}$

(d) $\frac{\sqrt{2}}{12}$

10. The value of the integral $\int_0^4 \int_{y/2}^2 e^{x^3} y\ dx\ dy$ is:

(a) $-\frac{2}{3}(e^8 + 1)$

(b) $\frac{2}{3}(e^8 - 1)$

(c) $\frac{2}{3}e^{64}$

(d) $\frac{2}{3}e^{64} - \frac{2}{3}$

11. The value of the integral $\int\int_D \cos(x^2 + y^2)\ dx\ dy$, where D is the region defined by $x^2 + y^2 \le 1$, is:

(a) cannot be evaluated

(b) $\pi \sin 1$

(c) $2\pi \sin 1$

(d) π

12. The value of the integral $\int\int\int_W \sqrt{x^2 + y^2 + z^2}\ dx\ dy\ dz$, where W is the solid bounded by $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$, is:

(a) 65π

(b) $5\pi^2$

(c) $\frac{76}{3}\pi$

(d) 10π

13. The work done by the force field $F(x, y, z) = x\ \mathbf{i} + y\ \mathbf{j}$ when a particle is moved along the path $(3t^2, t, 1)$ from $(12, 2, 1)$ to $(3, 1, 1)$ is equal to:

(a) 69

(b) 70

(c) -70

(d) -69

14. Consider the surface given by $(3\cos\theta\sin\phi, 2\sin\theta\sin\phi, \cos\phi)$, $\theta \in [0, 2\pi]$ and $\phi \in [0, \pi]$. An equation for the tangent plane to this surface at the point $(\frac{3}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})$ is given by:

(a) $x + y = -\frac{4}{\sqrt{2}}$

(b) $x + \frac{3}{2}y + z = 3\sqrt{2}$

(c) $x + 3z = 3\sqrt{2}$

(d) $y = 0$

15. The area of the cone $z^2 = x^2 + y^2$ lying in the region of space defined by $x \geq 0$, $y \geq 0$ and $0 \leq z \leq 1$ is:

(a) $\frac{\sqrt{2}\pi}{2}$

(b) $\frac{\sqrt{2}\pi}{4}$

(c) $\frac{\pi}{4}$

(d) $\sqrt{2}$

16. The surface integral $\int\int_S F \cdot d\mathbf{S}$, where $F = \mathbf{i} + \mathbf{j} + z(x^2 + y^2)^2\mathbf{k}$ and S is the surface $x^2 + y^2 = 1$, $0 \leq z \leq 1$, is equal to:

(a) π

(b) $\frac{14}{45}$

(c) 0

(d) 2π

17. The value of $\int_C (2x^3 - y^3)dx + (x^3 + y^3)dy$, where C is the positively oriented unit circle centered at the origin, is:

(a) $\frac{3\pi}{2}$

(b) 0

(c) $\frac{1}{2}$

(d) π

18. Let D be a region in the plane and ∂D the positively oriented boundary of D. Which of the following expressions does not represent the area of D?

(a) $\int_{\partial D}(y^2 - 1)dx + (xy^2)dy$

(b) $\int_{\partial D}(\frac{y^3}{3} - y)dx + (xy^2)dy$

(c) $\int_{\partial D} x\ dx + (x + y)dy$

(d) $\int_{\partial D} x\ dy$

19. The value of $\int\int_S (\nabla \times F) \cdot d\mathbf{S}$, where S is $x^2 + y^2 + z^2 = 9$, $z \geq 0$ with normal in the positive z direction and $F = x\mathbf{i}$, is:

(a) 0

(b) $9\pi + \frac{9}{2}$

(c) 9π

(d) 18π

20. The flux of $F = 3xy^2\mathbf{i} + 3x^2y\mathbf{j} + z^3\mathbf{k}$ out of the sphere $x^2 + y^2 + z^2 = 1$ is equal to:

(a) $\frac{6\pi}{5}$

(b) $\frac{12\pi}{5}$

(c) $\frac{4\pi}{3}$

(d) 0