Math 13 Worksheet #8: Change of variables and the Jacobian

(1) Use the transformation $x = \frac{1}{4}(u+v)$, $y = \frac{1}{4}(v-3u)$ to evaluate the integral

$$\iint_R (4x + 8y)dA,$$

where R is the parallelogram with vertices $(-1, 3)$, $(1, -3)$, $(3, -1)$, and $(1, 5)$.

(2) By making an appropriate change of variables, evaluate the integral $\iint_R \sin(9x^2+4y^2)dA$, where R is the region in the first quadrant bounded by the ellipse $9x^2+4y^2=1$.