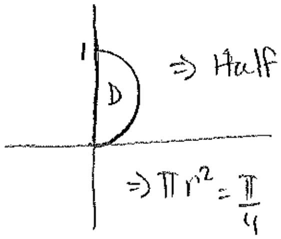
Double angle formulas
$\cos 2x = \cos^2 x - \sin^2 x$
$= 2\cos^2 x - 1$
$= 1 - 2\sin^2 x$

Math 13 Worksheet #3: Double integrals in polar coordinates

(1) Use polar coordinates to sketch the region and evaluate the expressions.

$$2\int_0^{\pi/2}\int_0^{\sin\theta} r\,dr\,d\theta$$



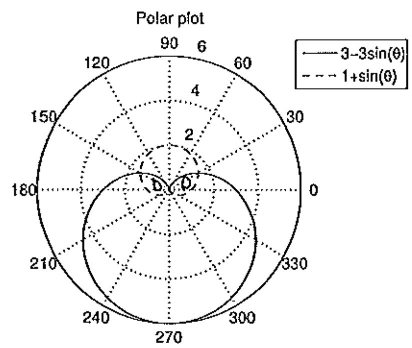
$\Rightarrow$ Half circle $\Rightarrow \pi r^2 = \frac{\pi}{4}$

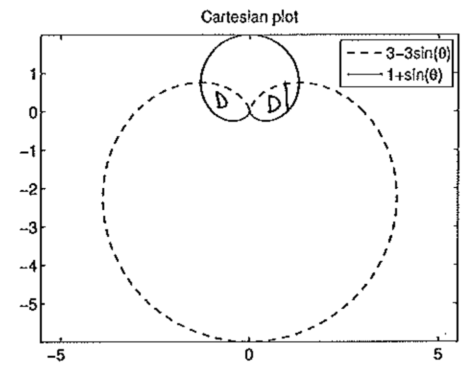
$= 2\int_0^{\pi/2} \frac{r^2}{2}\Big|_0^{\sin\theta} d\theta = 2\int_0^{\pi/2} \frac{\sin^2\theta}{2}\,d\theta$

$= \int_0^{\pi/2} \frac{1}{2}(1-\cos 2\theta)\,d\theta = \frac{\theta}{2} - \frac{\sin 2\theta}{4}\Big|_0^{\pi/2}$

$= \pi/4$ ✓

(2) Find the area of the region inside the cardioid $r = 3 - 3\sin\theta$ and outside the cardioid $r = 1 + \sin\theta$.





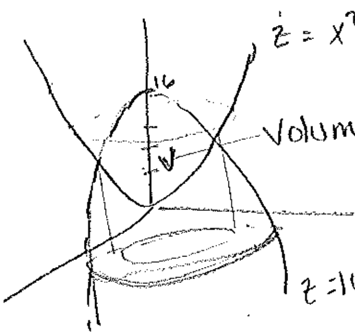
1st find intersections
$\rightarrow 3 - 3\sin\theta = 1 + \sin\theta$
$\rightarrow 2 = 4\sin\theta$
$\Rightarrow \sin\theta = 1/2 \Rightarrow \theta = \pi/6, 7\pi/6$.
only need 1.

$2\int_0^{\pi/6}\int_{1+\sin\theta}^{3-3\sin\theta} r\,dr\,d\theta$

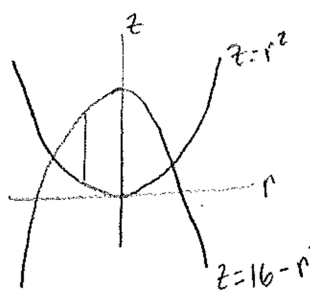
$= \frac{2}{2}\int_0^{\pi/6}\left[(3-3\sin\theta)^2 - (1+\sin\theta)^2\right]d\theta$

$= 8(\theta + \cos\theta)\Big|_0^{\pi/6} = 8\left[\frac{\pi}{6} + \cos(\pi/6) - 1\right]$

(3) Find the volume of the region enclosed by the paraboloids $z = x^2+y^2$ and $z = 16-x^2-y^2$.



$\rightarrow$



$V = \int_0^{2\pi}\int_0^{2\sqrt{2}} \left[(16-r^2)-(r^2)\right] r\,dr\,d\theta$

$= \int_0^{2\pi}\int_0^{2\sqrt{2}} \left[16-2r^2\right] r\,dr\,d\theta$

$= 64\pi.$

Intersection.
$r^2 = 16 - r^2 \Rightarrow r^2 = 8$
$r = 2\sqrt{2}$