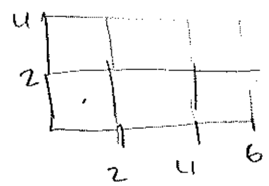
## Math 13 Worksheet #1: Integrating over rectangular regions

(1) Use the Midpoint Rule to estimate the volume of the solid that lies below the surface $z = xy$ and above the rectangle $R = \{(x, y) \mid 0 \le x \le 6, 0 \le y \le 4\}$. Let $m = 3$ and $n = 2$.




$$V \approx \Delta A\left[ f(1,1) + f(1,3) + f(3,1) + f(3,3) + f(5,1) + f(5,3) \right]$$

$$= 4[1 + 3 + 3 + 9 + 5 + 15]$$

$\Delta A = 4$

(2) Calculate the iterated integral.

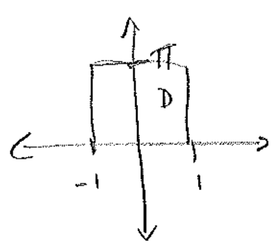
$$\int_0^1 \int_0^3 e^{x+3y}\,dx\,dy = \int_0^3 e^x\,dx \int_0^1 e^{3y}\,dy$$

$$= \left(e^x\Big|_0^3\right)\left(\frac{1}{3}e^{3y}\Big|_0^1\right) = (e^3 - 1)\,\frac{1}{3}(e^3 - 1)$$

Other way: $\int_0^3 e^x \int_0^1 e^{3y}\,dy\,dx = \int_0^3 e^x\left[\frac{1}{3}e^{3y}\Big|_0^1\right]dy = \frac{1}{3}\int_0^3 e^x(e^3 - 1)\,dx = \frac{1}{3}(e^3 - 1)\int_0^3 e^x\,dx$

$= \frac{1}{3}(e^3 - 1)(e^3 - 1)$ ✓

(3) Find the volume of the solid enclosed by the surface $z = 1 + e^x \sin y$ and planes $x = \pm 1$, $y = 0$, $y = \pi$, and $z = 0$.




$$\int_0^\pi \int_{-1}^1 (1 + e^x \sin y)\,dx\,dy$$

$$= \int_0^\pi x + e^x \sin y\Big|_{-1}^1 dy$$

$$= \int_0^\pi \left[1 + e\sin y + 1 - e^{-1}\sin y\right]dy$$

$$= \left[2y - e\cos y + e^{-1}\cos y\right]\Big|_0^\pi$$

$$= 2\pi - e\cos\pi + e^{-1}\cos\pi - (0 - e\cos 0 + e^{-1}\cos 0)$$

$$= 2\pi + e - e^{-1} + e - e^{-1} = 2\pi + 2e - 2e^{-1}$$