Math 13 Worksheet #17: Surface integrals of vector fields

(1) True or false:

(a) If $\boldsymbol{F}(x, y, z)$ is defined on an open region containing a smooth surface S, then $\int_S \boldsymbol{F}(x, y, z) \cdot \boldsymbol{n} dS$ measures the flow through the surface S in the direction $\boldsymbol{n}$ determined by the field $\boldsymbol{F}$.

(b) In computing $\int_S \boldsymbol{F} \cdot \boldsymbol{n} dS$, the direction of the normal vector is irrelevant.

(c) In computing $\int_S \boldsymbol{F} \cdot \boldsymbol{n} dS$ with $\boldsymbol{n}$ pointing in the correct direction, we could use a scalar multiple of $\boldsymbol{n}$, since the length will cancel in the dS term.

(2) Find the flux of $\boldsymbol{F}(x, y, z) =< -xz, -yz, z^2 >$ through the surface S where S is the cone with equation $z = \sqrt{x^2 + y^2}$ between $z = 2$ and $z = 4$ with $\boldsymbol{n}$ pointing outward.

(3) Find the flux of $\boldsymbol{F}(x, y, z) =< xz, 5z, y^2 >$ through the surface S where S is the region of the plane $12x - 9y + 3z = 20$, where $2 \leq x \leq 3$ and $5 \leq y \leq 10$, with $\boldsymbol{n}$ pointing upward.

(4) Suppose an electric field is given by $\boldsymbol{E}(x, y, z) =< 2y, 2xy, yz >$. Compute the flux $\int_S \boldsymbol{E} \cdot \boldsymbol{n} dA$ of the field through the unit cube $[0, 1] \times [0, 1] \times [0, 1]$.