Math 13 Worksheet #14: Curl and divergence

(1) Find the curl and divergence of $F(x,y,z) = < e^x \sin y, e^y \sin z, e^z \sin x >$.

$$\text{curl}\,\bar{F} = \nabla \times \bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial_x & \partial_y & \partial_z \\ e^x \sin y & e^y \sin z & e^z \sin x \end{vmatrix} = \bar{i}(0 - e^y \cos z) - \bar{j}(e^z \cos x - 0) + \bar{k}(0 - e^x \cos y)$$

$$= -e^y \cos z\,\bar{i} - e^z \cos x\,\bar{j} - e^x \cos y\,\bar{k}$$

$$\text{div}\,F = e^x \sin y + e^y \sin z \; e^z \sin x$$

(2) Determine if the vector field $F(x,y,z) = < 1, \sin z, y \cos z >$ is conservative. If it is, find a function f such that $\nabla f = F$.

$$\text{curl}\,\bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial_x & \partial_y & \partial_z \\ 1 & \sin z & y \cos z \end{vmatrix} = \bar{i}(\cos z - \cos z) - \bar{j}(0 - 0) + \bar{k}(0 - 0)$$

$= <0, 0, 0> \Rightarrow F$ is conservative.

$$f(x,y,z) = x + y \cos z + \underset{\text{constant}}{k}$$

(3) Determine if the vector field $F(x,y,z) = < 3xy^2z^2, 2x^2yz^2, x^2y^2z >$ is conservative. If it is, find a function f such that $\nabla f = F$.

$$\text{curl}\,\bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial_x & \partial_y & \partial_z \\ 3xy^2z^2 & 2x^2yz^2 & x^2y^2z \end{vmatrix} = \bar{i}(4xyz^2 - 6xyz^2) - \bar{j}(2xy^2z - 6xy^2z) + \bar{k}(4xyz^2 - 6xyz^2)$$

$\neq \bar{0} \Rightarrow \bar{F}$ is not conservative.

(4) Show that any vector field of the form $F(x,y,z) = f(y,z)i + g(x,z)j + h(x,y)k$ is incompressible.

4) imcompressible $\Rightarrow \nabla \cdot \bar{F} = 0$.

$$\nabla \cdot \bar{F} = \frac{d}{dx}(f(y,z)) + \frac{d}{dy}(g(x,z)) + \frac{d}{dz}(h(x,y))$$

$$= 0 \checkmark$$