Math 13 Worksheet #10: Fundamental Thm for Line Integrals

For the following problems, use the Fundamental Thm for line integrals, if applicable, to evaluate $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$. Otherwise show that the vector field is not conservative.

(1) $\boldsymbol{F}(x, y, z) = < -z, 1, x >$ with C a circular helix given by $x = \cos t$, $y = t$, and $z = \sin t$, for $0 \leq t \leq 2\pi$.

If $\bar{F} = <P, Q, R>$, $P_y = Q_x$ $\quad P_z = R_x$ & $Q_z = R_y$ $\Rightarrow$ conservative.

$P_y = 0 \stackrel{?}{=} Q_x = 0$ $\quad -1 = 1$

Not true $\Rightarrow$ Field is not conservative.

(2) $\boldsymbol{F}(x, y, z) = < yz^{xy} \ln z, xz^{xy} \ln z, \frac{z^{xy}}{z} >$ with C any curve from $(0, 0, 1)$ to $(2, 16, 3)$.

The field is conservative $f(x,y,z) = z^{xy}$

Then

$$\int_C \bar{F} \cdot d\bar{r} = \int_C \nabla f \cdot d\bar{r} = f(2,16,3) - f(0,0,1)$$
$$= 3^{32} - 1$$

(3) $\boldsymbol{F}(x, y, z) = < yze^{xyz} + 2, xze^{xyz} - 1, xye^{xyz} >$ and C is the curve of intersection of the surface $z = \sqrt{x^2 + y^2}$ and the plane $z - x + y = 10$.

the field is conservative

① $\quad f = \int P\,dx = \int (yze^{xyz} + 2)\,dx = e^{xyz} + 2x + g(y,z)$

Take derivative of ① $\quad f_y = xze^{xyz} + g_y = xze^{xyz} - 1 = R$

$\Rightarrow g_y(y,z) = -1 \Rightarrow g(y,z) = -y + h(z)$.

So $f(x,y,z) = e^{xyz} + 2x - y + h(z)$

$f_z = xye^{xyz} + h'(z) = xye^{xyz} \Rightarrow h'(z) = 0 \Rightarrow h(z) = K$

constant.

So $f(x,y,z) = e^{xyz} + 2x - y + K$

C is the intersection of the cone $z = \sqrt{x^2+y^2}$
& the plane $z - x + y = 10$

This will be an ellipse $\Rightarrow$ C is a closed curve

$\Rightarrow \int_C \bar{F} \cdot d\bar{r} = 0.$ since C closed & $\bar{F}$ conservative.