Math 13: Written Homework # 5
Due April 30 at 5pm

Please make sure your homework is stapled, if necessary before handing it in. Do not use paper clips or any variation of folding techniques to connect papers.

Solutions should be justified in a rigorous way. If you are unsure how much work to show, you can ask me prior to turning in your assignment. The problems are taken from the 7th edition of Stewart's *Calculus*, although occasionally a problem will be modified to be slightly different from its textbook counterpart.

(1) (Problem #16, Chapter 16.2) Evaluate the line integral $\int_C (y+z)dx + (x+z)dy + (x+y)dz$, where C is the concatenation of the line segment from $(0,0,0)$ to $(1,0,1)$ with the line segment from $(1,0,1)$ to $(0,1,2)$.

(2) (Problem #42, Chapter 16.2) The force exerted by a unit electric charge at the origin on a charged particle at a point (x,y,z) is $\boldsymbol{F}(\boldsymbol{r}) = \frac{\boldsymbol{r}}{|\boldsymbol{r}^3}$, where $\boldsymbol{r} =< x, y, z >$. (The textbook multiplies this equation by a constant K, but in this problem just assume $K = 1$.) Find the work done as the particle moves along a straight line from $(2,0,0)$ to $(2,1,5)$.

(3) (Problem #52, Chapter 16.2) Experiments show that a steady current I in a long wire produces a magnetic field B that is tangent to any circle that lies in the plane perpendicular to the wire and whose center is the axis of the wire (consult book for a picture). Amprére's Law relates the current to its magnetic effects and states that

$$\int_C \boldsymbol{B} \cdot d\boldsymbol{r} = \mu_0 I$$

where I is the net current that passes through any surface bounded by a closed curve C, and μ_0 is a constant called the permeability of free space. By taking C to be a circle with radius r, show that the magnitude $B = |\boldsymbol{B}|$ of the magnetic field at a distance r from the center of the wire is

$$B = \frac{\mu_0 I}{2\pi r}.$$

(4) (Problem #29, Chapter 16.3) Show that if the vector field $\boldsymbol{F} =< P, Q, R >$ is conservative and P, Q, and R have continuous first-order partial derivatives, then

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, \quad \frac{\partial P}{\partial z} = \frac{\partial R}{\partial x}, \quad \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}.$$

(5) (Problem #14, Chapter 16.3) Find a potential function $f(x,y)$ for $\boldsymbol{F} = \langle((1+xy)\,e^{xy},\ x^2 e^{xy}\rangle$, and evaluate $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$, where C is given by $\boldsymbol{r}(t) =< \cos t,\ 2\sin t >$, $0 \le t \le \pi/2$.

(6) (Problem #36a, Chapter 16.3) Suppose $\boldsymbol{F}$ is an inverse square field; that is,

$$\boldsymbol{F}(\boldsymbol{r}) = \frac{c\boldsymbol{r}}{|\boldsymbol{r}|^3}$$

for some constant c, where $\boldsymbol{r} =< x, y, z >$. Find the work done by $\boldsymbol{F}$ in moving an object from point P_1 along a path to a point P_2 in terms of the distances d_1, d_2 from these points to the origin.