Math 13: Written Homework # 2
Due April 9 at 5pm

Please make sure your homework is stapled, if necessary before handing it in. Do not use paper clips or any variation of folding techniques to connect papers.

Solutions be justified in a rigorous way. If you are unsure how much work to show, you can ask me prior to turning in your assignment.

(1) (Problem #13, Chapter 15.5) The boundary of a lamina consists of the semicircles $y = \sqrt{1-x^2}$ and $y = \sqrt{4-x^2}$ together with the portion of the $x-$axis that join them. Find the center of mass of the lamina if the density at any point is proportional to its distance from the origin.

(2) (Problem #28, Chapter 15.7) Sketch the solid whose volume is given by the following iterated integral, and compute the value of that volume

$$\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dxdzdy.$$

(3) (Problem #33, Chapter 15.7) (Consult the textbook for a useful figure.) The figure shows the region of integration for the integral

$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z)dzdydx.$$

Rewrite this integral as an equivalent iterated integral in the five other orders. Give some reasons for why your answers are correct (in particular, give a brief explanation of how you calculate projections of this region to the various coordinate planes).

(4) (Problem #24, Chapter 15.8) Find the volume of the solid that lies between the paraboloid $z = x^2 + y^2$ and the sphere $x^2 + y^2 + z^2 = 2$.

(5) (Problem #28, Chapter 15.8) Find the mass of a ball B given by $x^2 + y^2 + z^2 \leq a^2$ if the density at any point is proportional to its distance from the $z-$axis. Your density function will have a constant in it; you can either keep the constant in your calculations or just assume the constant is 1. (Think about why this problem is better suited to cylindrical than spherical coordinates; you don't have to answer this in writing, but it is worth understanding why you use cylindrical coordinates here.)

(6) (Problem #28, Chapter 15.9) Find the average distance from a point in a ball of radius a to its center.