

1. Let $\mathbf{F}(x, y, z) = \langle 2x^2y, 3xy^2, -zxy \rangle$. What is $\nabla \cdot \mathbf{F}$?

$$\begin{aligned}\nabla \cdot \mathbf{F} &= \left\langle \frac{\delta}{\delta x}, \frac{\delta}{\delta y}, \frac{\delta}{\delta z} \right\rangle \cdot \langle 2x^2y, 3xy^2, -zxy \rangle = \\ &\frac{\delta}{\delta x} 2x^2y + \frac{\delta}{\delta y} 3xy^2 + \frac{\delta}{\delta z} -xyz = 4xy + 6xy - xy = 9xy\end{aligned}$$

2. What does your answer to the above [and the appropriate theorem] indicate regarding the relationship of $\iiint_V -3xy \, dV$ and $\iint_S \langle 2x^2y, 3xy^2, -zxy \rangle \cdot d\mathbf{S}$ where V is the [positively oriented] unit ball and S is the [positively oriented] unit sphere?

Since $9xy = -3(3xy)$ we should have that

$$\int \int_{\partial V} \langle 2x^2y, 3xy^2, -zxy \rangle \cdot d\mathbf{S} = -3 \int \int \int_V -3xy \, dV$$

3. Is there a vector function \mathbf{G} such that $\mathbf{F} = \nabla \times \mathbf{G}$? [Where \mathbf{F} is the \mathbf{F} in the first problem.] Why?

Since $\nabla \cdot \mathbf{F} \neq 0$ we cannot have \mathbf{G} such that $\mathbf{F} = \nabla \times \mathbf{G}$ since then we would have $9xy = \nabla \cdot \mathbf{F} = \nabla \cdot (\nabla \times \mathbf{G}) = 0$, which is not true in general.