Fundamental Theorem of Line Integrals.

- If we remember that $g = \frac{df}{dt}$ meant that $\int_b^a g dt = f(b) - f(a)$ it makes sense in a vector setting that $\mathbf{F} = \nabla f$ implies $\int_b^a \mathbf{F} \cdot \mathbf{r}'(\mathbf{t}) dt = f(\mathbf{r}(\mathbf{b})) - f(\mathbf{r}(\mathbf{t}))$.

- f is related to the "potential energy" caused by the force F. It is actually the negative of it.

- To decide if a vector function $\mathbf{F}$ is the gradient of some other function, calculate the curl of $\mathbf{F}$. In particular if $\nabla \times \mathbf{F} = \mathbf{0}$ then there exists some function f such that $\mathbf{F} = \nabla f$.

- if $\mathbf{F} = \nabla f$ then $\int_a^b \mathbf{F}(\mathbf{r}(\mathbf{t})) \cdot \mathbf{f}'(\mathbf{t}) dt = f(\mathbf{r}(\mathbf{b})) - f(\mathbf{r}(\mathbf{a}))$...in particular it does not depend on **path** but only the end-points.

Curl

- If we think of ∇ as a 'vector' in the sense that $\nabla = \frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}}$ then the curl can be calculated as $\nabla \times \mathbf{F}$,the cross product of the 'vector' ∇ and the vector-function $\mathbf{F} = f_1\hat{\mathbf{i}} + f_1\hat{\mathbf{j}} + f_3\hat{\mathbf{k}}$ [note that the f_1, f_2, f_3 have nothing to do with f [where $\mathbf{F} = \nabla \times f$, but are merely the components of $\mathbf{F}$...so if $\mathbf{F} = \langle x^2, xy, y^2z\rangle$, then $f_1 = x^2$, $f_2 = xy$, $f_3 = y^2z$.]