Scalar Line Integral [Path Integral]

- when path is parametrized by arc length, we have a natural analog of the integral done earlier in 1 dimension. In fact we have that the integral of a scalar function f along a curve $r(s)$ is simply $\int f(r(s))ds$.

- The above does not work for all parametrizations because the speed may not be constantly 1. [We think of the earlier integrals as integrals on the x-axis with the parametrization being $r(t) =< t, 0, 0 >$, which has speed 1].

- For arbitrary parametrizations $r(t)$ we can calculate the scalar line integral as $\int f(r(s))ds = \int f(r(t))\frac{ds}{dt}dt = \int f(r(t)) \mid r'(t) \mid dt$.

- The most obvious application of a scalar line integral is to find the mass of an object whose density fluctuates...in particular if μ is **linear density** we have $m = \int \mu(r(s))ds$.