Finding Area enclosed by a curve.

If a curve is in the $x-y$ plane, then, since $\mathbf{dS} = \langle 0, 0, |\mathbf{dS}| \rangle$ if $\nabla \times F = \langle 0, 0, 1 \rangle$ then we have

$$Area = \int \int_S dS = \int \int \langle 0, 0, 1 \rangle \cdot \mathbf{dS} =$$

$$\int \int_S \nabla \times F \cdot dS = \int_{\partial S} F \cdot dr$$

Possible choices for $\mathbf{F}$ are $\mathbf{F} = \langle -y, 0, 0 \rangle$ or $\mathbf{F} = \langle 0, x, 0 \rangle$ or $\mathbf{F} = \langle \frac{-y}{2}, \frac{x}{2}, 0 \rangle$