These questions are meant to build up your conceptual understanding of the material. They are due by Monday. You should look over this early so you can ask questions in class about the meaning of these questions.

1. What is the boundary of each of the following? VERY IMPORTANT: parameterize the boundary in a way that respects the orientation of the set involved.

- a. The paraboloid defined by $z = x^2 + y^2$ for $x^2 + y^2 \leq 4$.
- b The Cylinder $x^2 + y^2 = 10$, $0 \leq z \leq 5$ oriented so that the normal vector is out.
- c The line $r(t) = \langle t^2, t^2 \rangle, 1 \leq t \leq 4$.

2. For each of the sets, find a region that has the set as a boundary. VERY IMPORTANT: parameterize your answer in a way that respects the orientation given in the boundary.

- a The parallelogram with corners $\langle 0,0,2\rangle, \langle 2,1,5\rangle, \langle 4,4,4\rangle, \langle 6,5,7\rangle$ oriented in such a way that the normal vector points up [though not straight up].
- the points $\langle 3,2,1\rangle, \langle 4,4,4\rangle, \langle 2,4,5\rangle, \langle 0,3,9\rangle$ where the first 2 are considered positive and the last two are considered negative.

3. We discussed in class how to find the proper orientation for a line that is given as the boundary of an oriented surface. We also discussed the fine points of taking the "boundary" of a line segment, and how to orient that boundary. How would you figure out how to give the proper orientation to the boundary of an oriented volume?

4. Consider the region under the paraboloid $z = 4 - x^2 - y^2$ and above the plane $z = 1$. Call this region V. This looks like a hollow bullet. I can think of the volume as being bounded by the shell made of a disk and a parabolic dome [the nose of the bullet]. If I call the disk $S_1$and the nose S_2, and I parameterize S_1 by $\langle u, v\rangle \to \langle u(cos(v)), u(sin(v)), 1\rangle$ and parameterize S_2 by $\langle u, v\rangle \to \langle u(cos(v)), u(sin(v)), 4-u^2\rangle$ [for u and v having the appropriate bounds]. If $F = \langle x, y, -1\rangle$ I <u>should</u> be able to say the integral of $\nabla \cdot \mathbf{F}$ through the region V is equal to the surface integral of $\mathbf{F}$ on the boundary of V, that is S_1 plus S_2, and hence I should be able to do the integral of F on the two parameterized surfaces and sum them to get the same answer [as integrating $\nabla \dot{F}$ throughout V]. It doesn't work. Why not?

5. Consider the function $\mathbf{F(x,y,z)} = \langle \frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}\rangle$

Calculate $\nabla \cdot F$. You should get 0. If you do not, you may consider going back through your work until you do.

Now, if I let S be the sphere of radius 1, I should find the integral of $\mathbf{F}$ on the sphere is equal to the integral of $\nabla \cdot F$ in the unit ball. Since $\nabla \cdot F = 0$ this says the integral on the sphere is 0. However, it appears from looking at the function that the vector

$\langle \frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}} \rangle$ points out of the sphere at all points on the surface, and hence there *should*, one would think, be some non-trivial flux....that is the integral should not be zero. What is wrong?