Math 13 Fall 2009 practice final

1. Show that the ellipsoid $3x^2 + 2y^2 + z^2 = 9$ and the sphere $x^2 + y^2 + z^2 - 8x - 6y - 8z + 24 = 0$ are tangent at the point $(1, 1, 2)$. That is, show that this point is on each of the surfaces and that the tangent planes at $(1, 1, 2)$ of both are equal.

2. Compute the intersection of the two lines $c_1(t) = (0, -1, 2) + t\langle -1, 1, 2\rangle$ and $c_2(t) = (-1 + t, -t, 2 - t)$. Find the plane that contains the two lines.

3. Suppose that $f : \mathbb{R}^2 \to \mathbb{R}^2$ and $g : \mathbb{R}^2 \to \mathbb{R}^2$ are continuously differentiable functions and suppose you know the following facts about f and g:
$f(1, 2) = (3, 4)$
$Df(1, 2) = \begin{pmatrix} 2 & 6 \\ 3 & 7 \end{pmatrix}$ $\qquad Dg(1, 2) = \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix}$ $\qquad Dg(1, 2) = \begin{pmatrix} 4 & 8 \\ 1 & 2 \end{pmatrix}$
Find $D(g \circ f)(1, 2)$

4. Calculate the work done by the force field $F(x, y) = (5x^2y, e^y)$ on a particle that moves along the parabola $y = x^2$ from $(-1, 1)$ to $(2, 4)$.

5. Let c be the curve which is the boundary of the region between the parabola $x = y^2 - 4$, the line $x = 0$, and the line $y = 0$. Orient c so that you are traversing it clockwise. Let $F(x, y) = \langle e^x + x^2y, x^2\rangle$. Use Green's theorem to evaluate $\int_c (F \circ c) \cdot c' dt$.

6. Let $F(x, y, z) = z^2 i + xj + 0k$, and let S be the part of the surface $z = y^2$ lying between the planes $x = 0$ and $x = 1$ and below the plane $z = 1$. Give S the outward pointing normal (its k component is negative). Evaluate $\int_S F \cdot ndS$.

7. Use the divergence theorem to evaluate $\int_S F \cdot ndS$ where S is the part of the sphere $x^2 + y^2 + z^2 = 1$ with $z > 0$, oriented so that the normal is pointing outward (away from the origin) and $F(x, y, z) = \langle x, e^x, 1\rangle$.

8. Let $f(x, y) = \cos(x) \cos(2y)$ find the local maxima and minima of f for$x \in [0, \pi]$ and $y \in [-\frac{\pi}{2}, \frac{\pi}{2}]$. What is the tangent plane of f at the point $(0, 0)$.

9. Consider the regular (equilateral-symmetric) octogon which lies in the upper half plane (where $y > 0$) which has one of its sides on the x-axis from $(0, 0)$ to $(1, 0)$. Let C be the curve which tracces around the octogon from $(1, 0)$ to $(0, 0)$ counter clockwise. Calculuate $\int_C \mathbf{F} \cdot \mathbf{T} ds$ where $F(x, y) = (3x, 4x + 44y \sin(y^2))$.

10. Consider the curve C parameterized by $\mathbf{r}(t) = (0, 2 + 2\cos(t), 2 + 2\sin(t))$ on $[0, 2\pi]$. Let $\mathbf{F}(x, y, z) = (x^2 e^{5z}, x\cos(y), 3y)$. Calculuate $\int_C \mathbf{F} \cdot \mathbf{T} ds$.

11. If $\mathbf{F}$ is a conservative vector field with scalar potential function f, what is $\int_C \mathbf{F} \cdot \mathbf{T} ds$ if C goes from (x_1, y_1, z_1) to (x_2, y_2, z_2)?