Math 13 Fall 2004

Calculus of Vector-valued Functions

Example of a function that has both partial derivatives at (0,0), but is not differentiable

October 4, 2004

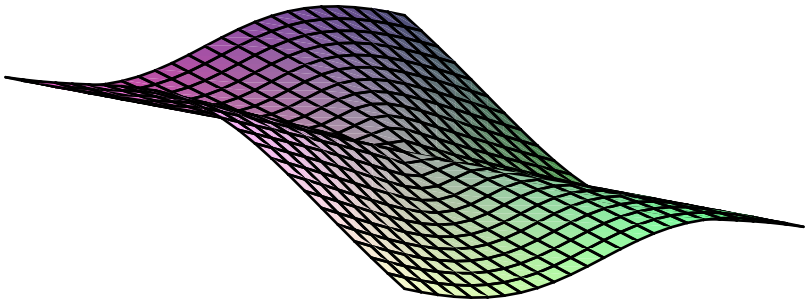
Define a scalar-valued function of two variables

```
> f := (x, y) -> x^3 / (x^2 + y^2);
```

$$f := (x, y) \to \frac{x^3}{x^2 + y^2}$$

Have a look at its graph

```
> plot3d(f(x, y), x = -1..1, y = -1..1);
```

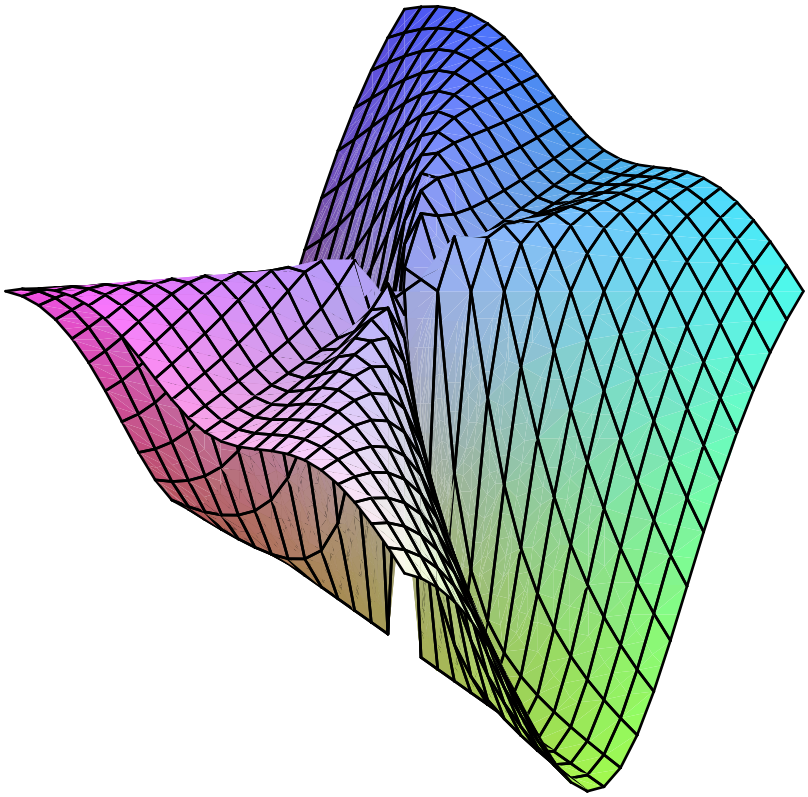


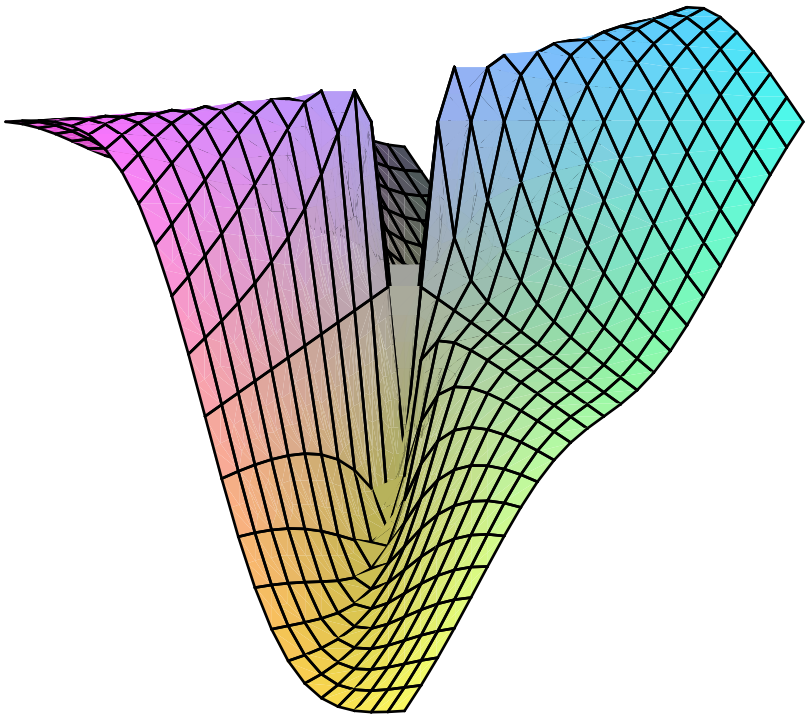
f seems to be continuous everywhere and indeed it is.

But f is not differentiable at (0, 0) (see the handout for details).

Let's plot its partial derivatives

```
> plot3d(diff(f(x, y), x), x = -1..1, y = -1..1);
  plot3d(diff(f(x, y), y), x = -1..1, y = -1..1);
```





They are obviously discontinuous!!!

```
> factor(diff(f(x, y), x));
  factor(diff(f(x, y), y));
```

$$\frac{x^2 (x^2 + 3 y^2)}{(x^2 + y^2)^2}$$

$$-\frac{2 x^3 y}{(x^2 + y^2)^2}$$

```
>
```