Average Values and Center of Mass

Average value of an one-variable function $f : [a,b] \to \mathbb{R}$:

$$[f]\text{avg} = \frac{\int_a^b f(x)dx}{b-a} = \frac{\int_a^b f(x)dx}{\int_a^b dx} = \frac{\int_a^b f(x)dx}{\text{length of } [a,b]}$$

Average value of a two-variable function $f : D \to \mathbb{R}$:

$$[f]\text{avg} = \frac{\iint_D f\,dA}{\iint_D dA} = \frac{\iint_D f\,dA}{\text{area of } D}$$

Average value of a three-variable function $f : W \to \mathbb{R}$:

$$[f]\text{avg} = \frac{\iiint_W f\,dV}{\iiint_W dV} = \frac{\iiint_W f\,dV}{\text{volume of } W}$$

Center of mass in $\mathbb{R}$: a wire between $x = a$ and $x = b$ with density per unit length $\delta(x)$

$$\bar{x} = \frac{\text{total moment}}{\text{total mass}} = \frac{\int_a^b x\,\delta(x)dx}{\int_a^b \delta(x)dx}$$

Center of mass in $\mathbb{R}^2$: a lamina represented by the region D with density per unit area $\delta(x,y)$

$$\bar{x} = \frac{\iint_D x\,\delta(x,y)\,dA}{\iint_D \delta(x,y)\,dA} \qquad \bar{y} = \frac{\iint_D y\,\delta(x,y)\,dA}{\iint_D \delta(x,y)\,dA}$$

Center of mass in $\mathbb{R}^3$: a solid W with density per unit volume $\delta(x,y,z)$

$$\bar{x} = \frac{\iiint_W x\,\delta(x,y,z)\,dV}{\iiint_W \delta(x,y,z)\,dV} \qquad \bar{y} = \frac{\iiint_W y\,\delta(x,y,z)\,dV}{\iiint_W \delta(x,y,z)\,dV} \qquad \bar{z} = \frac{\iiint_W z\,\delta(x,y,z)\,dV}{\iiint_W \delta(x,y,z)\,dV}$$