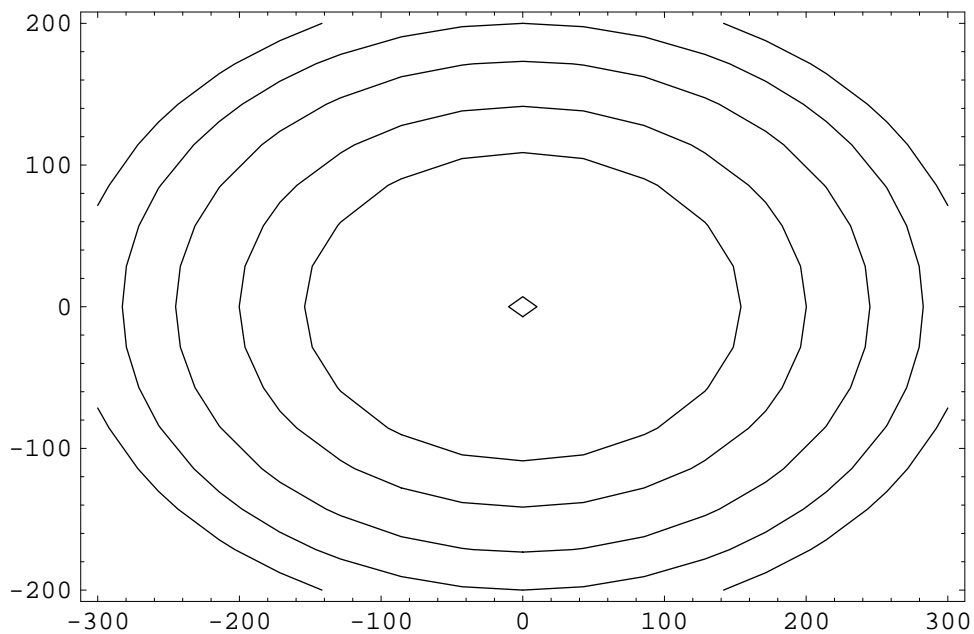


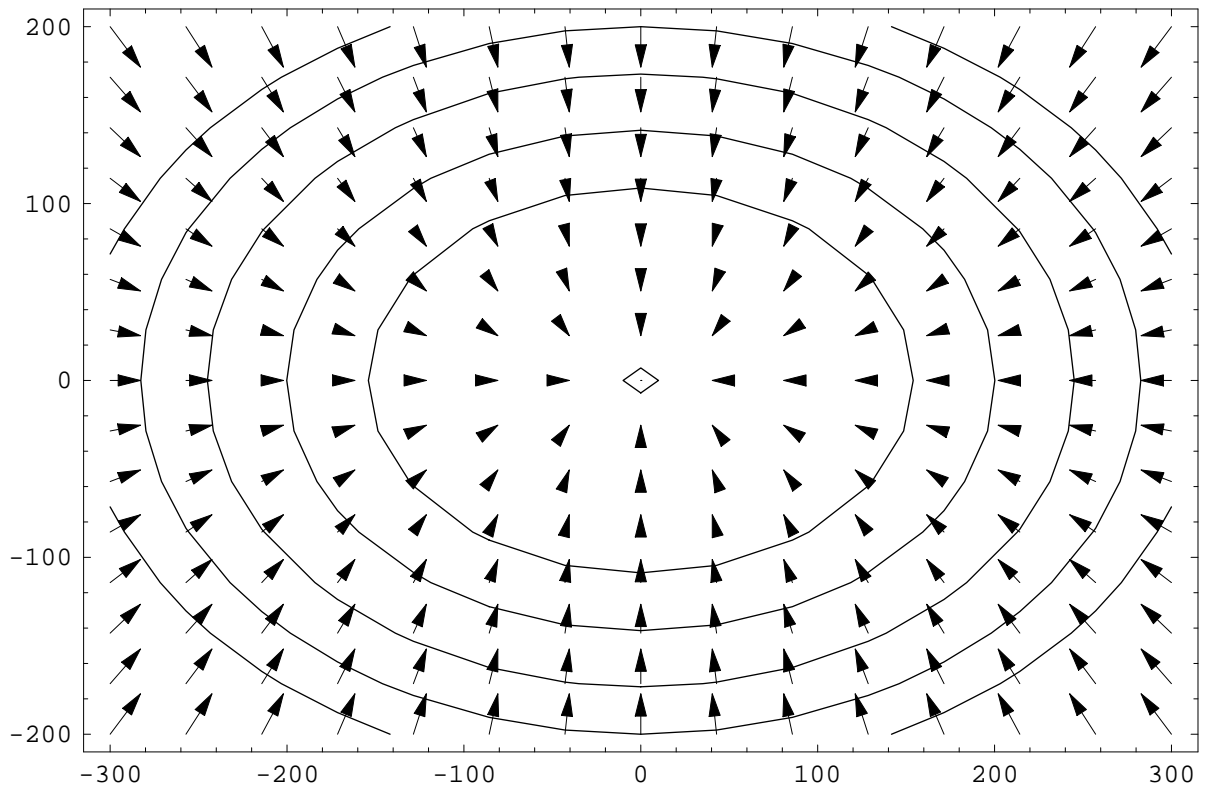
SECTION 2.5 THE GRADIENT VECTOR FIELD

You are climbing a hill whose shape is given by the equation
$$z = 1000 - 0.01x^2 - 0.02y^2$$
and you are standing at the point with coordinates $(60, 100, 764)$. In which direction should you proceed initially in order to reach the top of the hill fastest? Draw the level curves and the gradient.

Solution. The graph below contains the level curves $c=0,200,400,600,764,999$:



The next one represents also the gradient:



The picture shows clearly that the gradient is perpendicular to the level curves. The answer to the given problem comes now recalling another property that the gradient satisfies: it indicates the direction of maximum change for the function. Consequently one should follow the gradient to get fastest to the top of the hill.

```
** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** ** **  
EXAMPLE (to play with!) For the function :  
       $f(x, y) = x*y - 0.5*x^3 - 0.5*y^3$   
the graph and some level curves are given in the  
pictures below. Draw the gradient vector field.
```

