MATH 125: QUATERNION ALGEBRAS SPRING 2014

JOHN VOIGHT

Course Info

- Lectures: Monday, Wednesday, Friday, 10A (10:00–11:05 a.m.)
- **x-period**: Thursday, 12:00 noon–12:50 p.m.
- Dates: 24 January 2014–28 May 2014
- Room: 004 Kemeny
- Instructor: John Voight
- Office: Kemeny Hall, Room 240
- E-mail: jvoight@gmail.com
- Instructor's Office Hours: Monday 1:00–2:30 p.m., Wednesday 2:00–3:30 p.m., or just make an appointment!
- Course Web Page: http://www.math.dartmouth.edu/~jvoight/125/
- **Prerequisites**: Some advanced coursework and a healthy dose of curiosity!
- **Required Texts**: None (course notes on webpage).
- Grading: For undergraduates, grade will be based on weekly homework.

Homework

The homework assignments will be assigned on a varying basis and will be posted on the course webpage. Homework is required for undergraduates and optional but strongly encouraged for graduate students. In general, it is due in one week, but late homework will be accepted.

Cooperation on homework is permitted (and encouraged), but if you work together, do not take any paper away with you—in other words, you can share your thoughts (say on a blackboard), but you have to walk away with only your understanding. In particular, write the solution up on your own. Please write on your assignment the names of any other collaborators you worked with.

Plagiarism, collusion, or other violations of the Academic Honor Principle, after consultation, will be referred to the The Committee on Standards.

Religious Observances and Accommodation

Some students may wish to take part in religious observances that occur during this academic term. If you have a religious observance that conflicts with your participation in the course, please meet with me before the end of the second week of the term to discuss appropriate accommodations.

I encourage students with disabilities, including "invisible" disabilities such as chronic diseases and learning disabilities, to discuss with me after class or during my office hours appropriate accommodations that might be helpful to you.

Students with disabilities enrolled in this course and who may need disability-related classroom accommodations are encouraged to make an appointment to see me before the end of the second week of the term. All discussions will remain confidential, although the Student Accessibility Services office may be consulted to discuss appropriate implementation of any accommodation requested.

TOPICS

This course will introduce the arithmetic theory of quaternion algebras from a explicit point of view. Quaternion algebras lie at the crossroads of many areas of mathematics: number theory, Diophantine equations, group theory, noncommutative algebra, automorphic forms—even coding and network theory. We will touch on as many of these aspects as possible.

Topics include:

- Quaternion algebras over fields. Relationship to quadratic forms. Related basic algorithmic problems. Hilbert symbols and recognizing the matrix ring.
- Global arithmetic of quaternion orders (over Dedekind domains). Types of orders. Computing a maximal order.
- Local arithmetic of quaternion orders (over DVRs). Local embedding numbers.
- Ideal theory of orders over Dedekind domains. Ideal classes and their algorithmic enumeration. Global embedding numbers. Strong approximation.
- Unit groups of quaternion orders and geometry. Arithmetic Fuchsian groups. Fundamental domains. Shimura curves and Shimura varieties, explicit methods. Brandt matrices and automorphic forms. Ramanujan graphs.
- Other topics as time allows.



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