Mathematics 11
Practice Exam 2

1. Consider the function
$$f(x,y) = 2xy + x^2.$$
   (a) Find all critical points of $f$.
   (b) For each critical point, determine whether it is a local maximum, local minimum, or saddle point.
   (c) Consider the rectangular region
$$D = \{(x,y) \mid -1 \leq x \leq 1,\ -1 \leq y \leq 1\}.$$
   Determine the absolute maximum value of $f$ on $D$, and state the point(s) where $f$ attains this value.

2. Let
$$F(x,y) = \langle ye^{xy}\sin y + \cos y,\ xe^{xy}\sin y + e^{xy}\cos y\rangle.$$
   Is $F$ conservative?

   If so, find a potential function for $F$.

   If not, find a closed curve (a loop) $C$ for which $\int_C F \cdot d\vec{r} \neq 0$, and determine the value of $\int_C F \cdot d\vec{r}$.

3. Let $R$ be the rectangle with vertices at $(1,2)$, $(6,2)$, $(6,5)$, $(1,5)$, and let $C$ be the curve that traverses the sides of $R$ counterclockwise.

   Suppose $f(x,y)$ is a function on $\mathbb{R}^2$ satisfying $3 \leq f(x,y) \leq 7$ for all $x, y$. What is the maximum possible value of
$$\int_C f(x,y)dx + f(x,y)dy?$$

4. Find
$$\iiint_E x\, dV$$
   where $E$ is the region in $\mathbb{R}^3$ above the $xy$-plane, below the surface $z = 1 - x^2$, and between the planes $y = 0$ and $y = 4$.

5. Do NOT evaluate the following integrals.

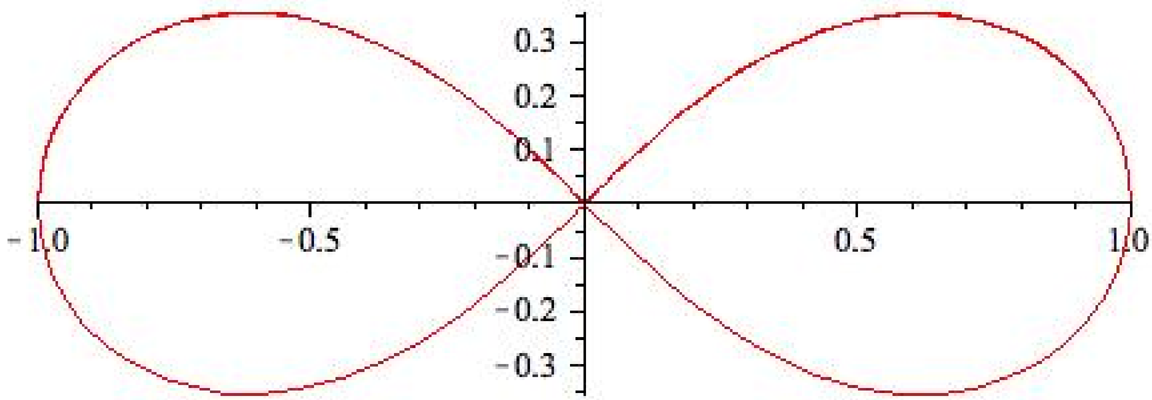


Figure 1: The lemniscate of Bernoulli (problem 6).

(a) Rewrite
$$\int_0^{\pi} \int_0^4 \int_{-r^2}^0 zr^4 \cos\theta \; dz \; dr \; d\theta$$
as an integral or sum of integrals in rectangular coordinates.

(b) Rewrite
$$\int_0^1 \int_{1-y}^1 x^2 + y^2 \, dx \, dy$$
as an integral or sum of integrals in polar coordinates.

6. The lemniscate of Bernoulli is a curve in the plane defined by the equation
$$(x^2 + y^2)^2 = x^2 - y^2$$
(see the picture). Let $R$ be the portion of the right petal of the lemniscate above the $x$-axis. Evaluate $\iint_R xy \, dA$.