Math 11
Fall 2012

Practice Exam I

This practice exam is intended to give you an idea of the kinds of problems we consider putting on an exam, and of the possible length of a midterm exam. Any topic that appeared on a homework problem may appear on the exam, as may anything covered in class or in the reading. You will notice that some of the following problems are not quite like any homework problem, but require you to think about what you have learned and apply it in new ways. Others are quite straightforward, and very similar to homework problems.

Here is a good way to use the practice exam to study. *First*, before you even look at the practice exam, study. *Then*, when you feel prepared to take the exam, take the practice exam in a single sitting in a quiet place. If you do well on these sample problems under those conditions, you are probably ready for the exam; if not, you will find some area or areas you should review more thoroughly.

Following the actual practice exam are some additional practice problems of the sort that could appear on an exam.

Disclaimer: A single exam, or practice exam, or review sheet, cannot possibly include every specific topic or problem that might be on an exam. There may be things on the actual exam that were covered in class, reading, or homework, but do not appear here.

1. (a) Find a equation for the plane containing the points $(-1, 0, 3)$, $(0, 1, 2)$, and $(1, 1, -1)$.
   (b) Find an equation for the plane parallel to the plane in part (a) and containing the point $(1, 1, 1)$.

2. (a) Show that the curve $\langle 2\sqrt{5}\cos\theta,\, 2\sqrt{5}\sin\theta,\, 4\rangle$ is the intersection of the sphere of radius 6 centered at the origin and the paraboloid $5z = x^2 + y^2$.
   (b) Find the arclength of the curve $\vec{r}(t) = \langle 7\sqrt{2}t,\, e^{7t},\, e^{-7t}\rangle$, $0 \le t \le 1$.

3. Find the limits or show they do not exist.

   (a)
$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + 2xy + y^2}$$

   (b)
$$\lim_{(x,y)\to(0,0)} \frac{x^4 + x^2y^2 - y^4}{x^2 + y^2}$$

   (c)
$$\lim_{(x,y)\to(1,2)} \frac{\cos(4x^2 + y^2 - 8) - 1}{4x^2 + y^2 - 8}$$

4. Let S be the surface $x + y = 2z^2$, and γ be the intersection of S with the plane $x = y$.

   (a) Find a function f such that S is a level surface of f, and find a vector $\vec{n}$ perpendicular to S at the point $(4, 4, 2)$.
   (b) Find a function $\vec{r}$ parametrizing the curve γ, and show directly (without using the fact that $\vec{n}$ is perpendicular to S) that at the point $(4, 4, 2)$, the vector $\vec{n}$ is perpendicular to the direction of γ. (Hint: Let $z = t$.)
   (c) Use the chain rule to compute $\dfrac{d}{dt} f(\vec{r}(t))$.

5. Use partial derivatives to approximate $\sqrt{81.2} - \sqrt[3]{124.8}$.

6. Each function matches exactly one of the pictures on the following pages, either a graph, or a set of level curves (for equally spaced values of f). Identify the picture that goes with each function.

   (a) $f(x, y) = x^2 + 4y^2$
   (b) $f(x, y) = x^2 + 2xy + y^2$
   (c) $f(x, y) = 4x^2 - 2y^2$

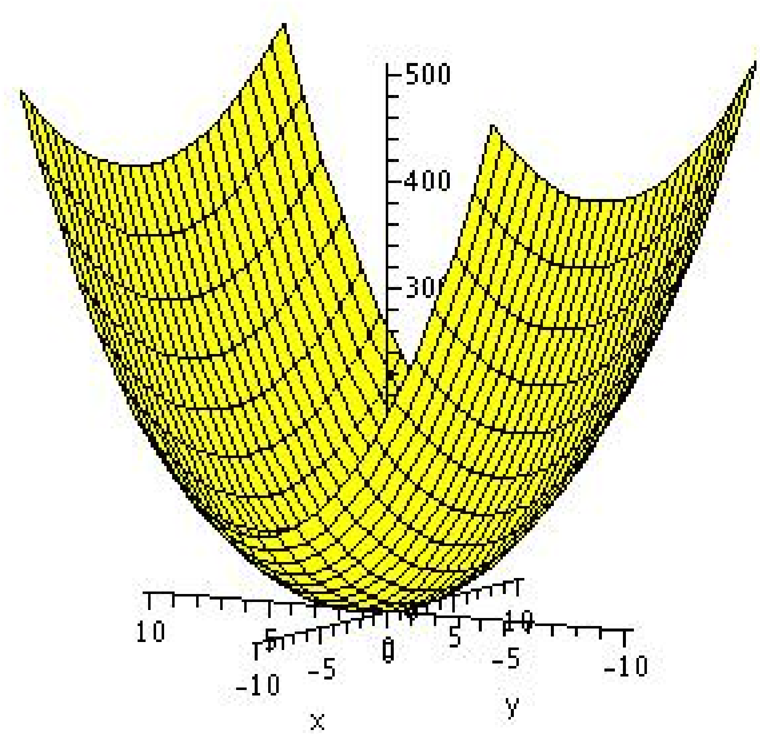

Figure 1: Graph of f.

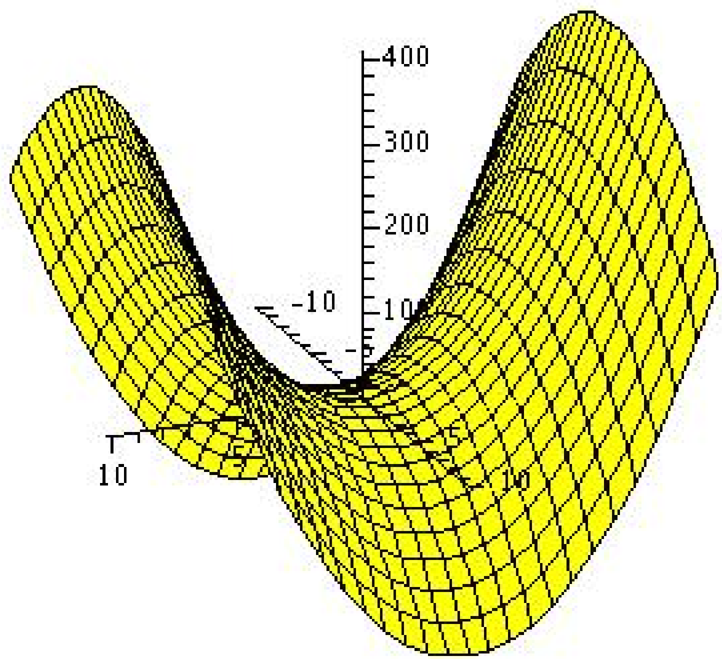

Figure 2: Graph of f.

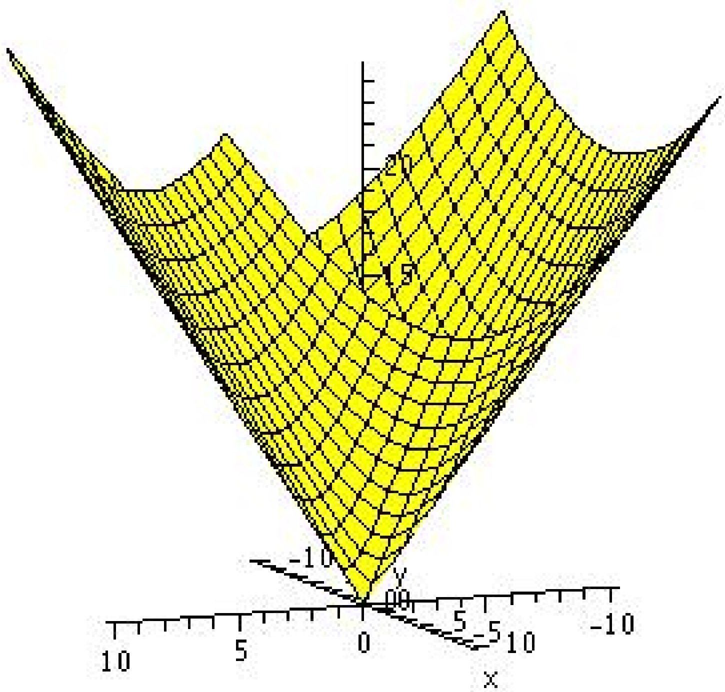

Figure 3: Graph of f.

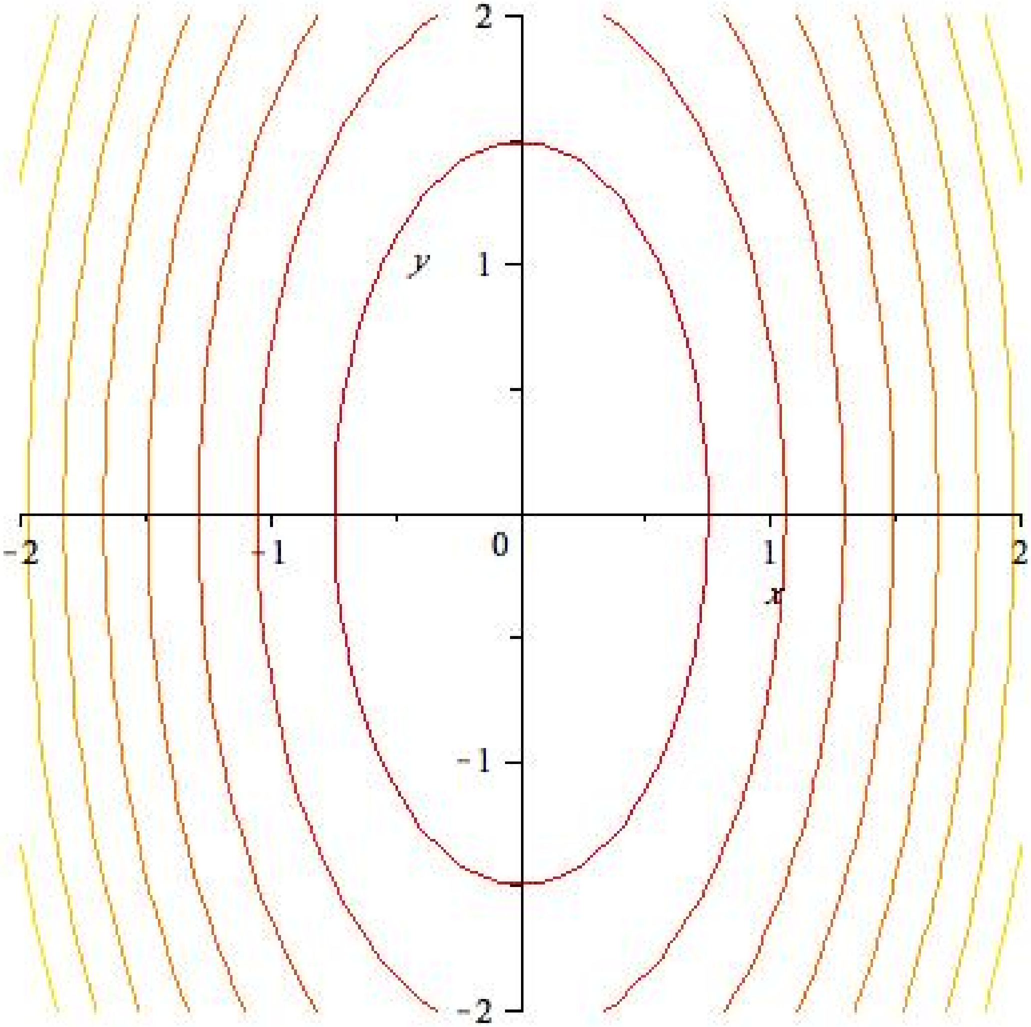

Figure 4: Level curves of f.

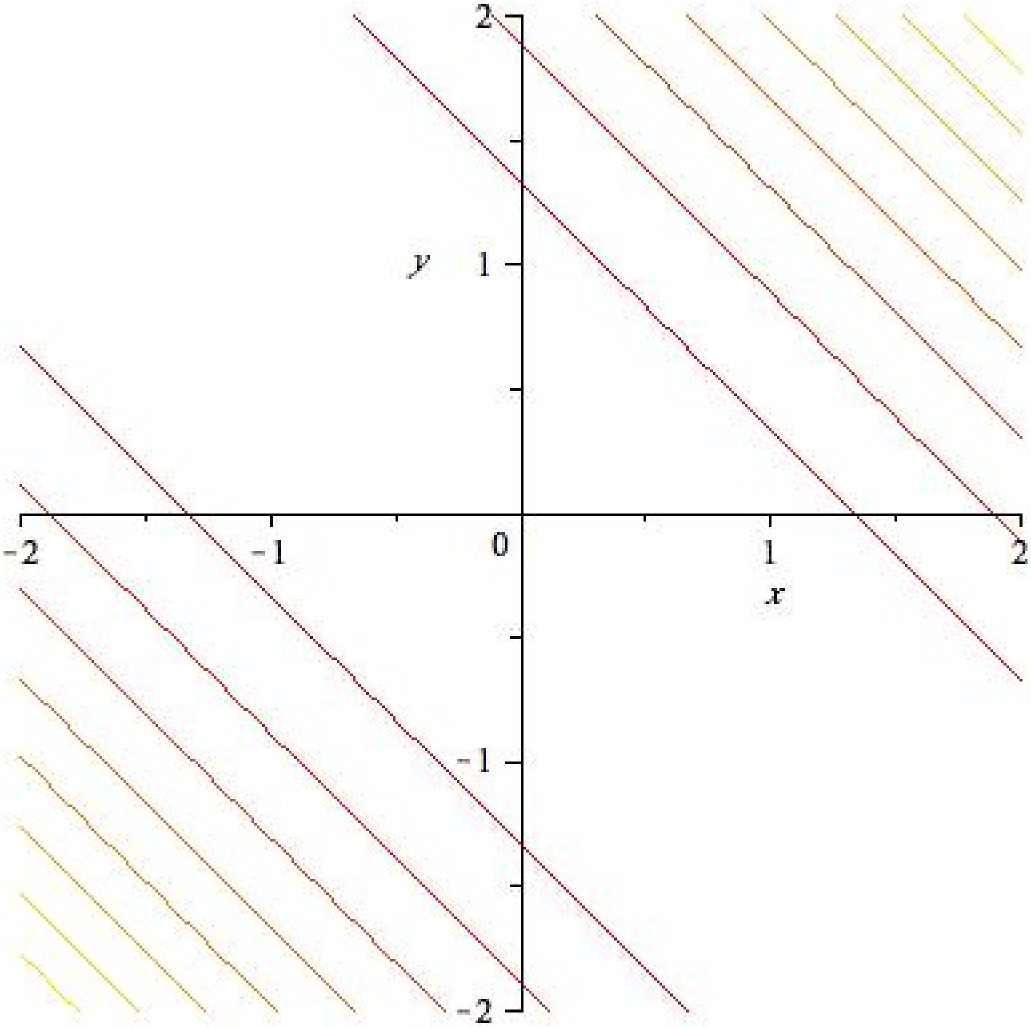

Figure 5: Level curves of f.

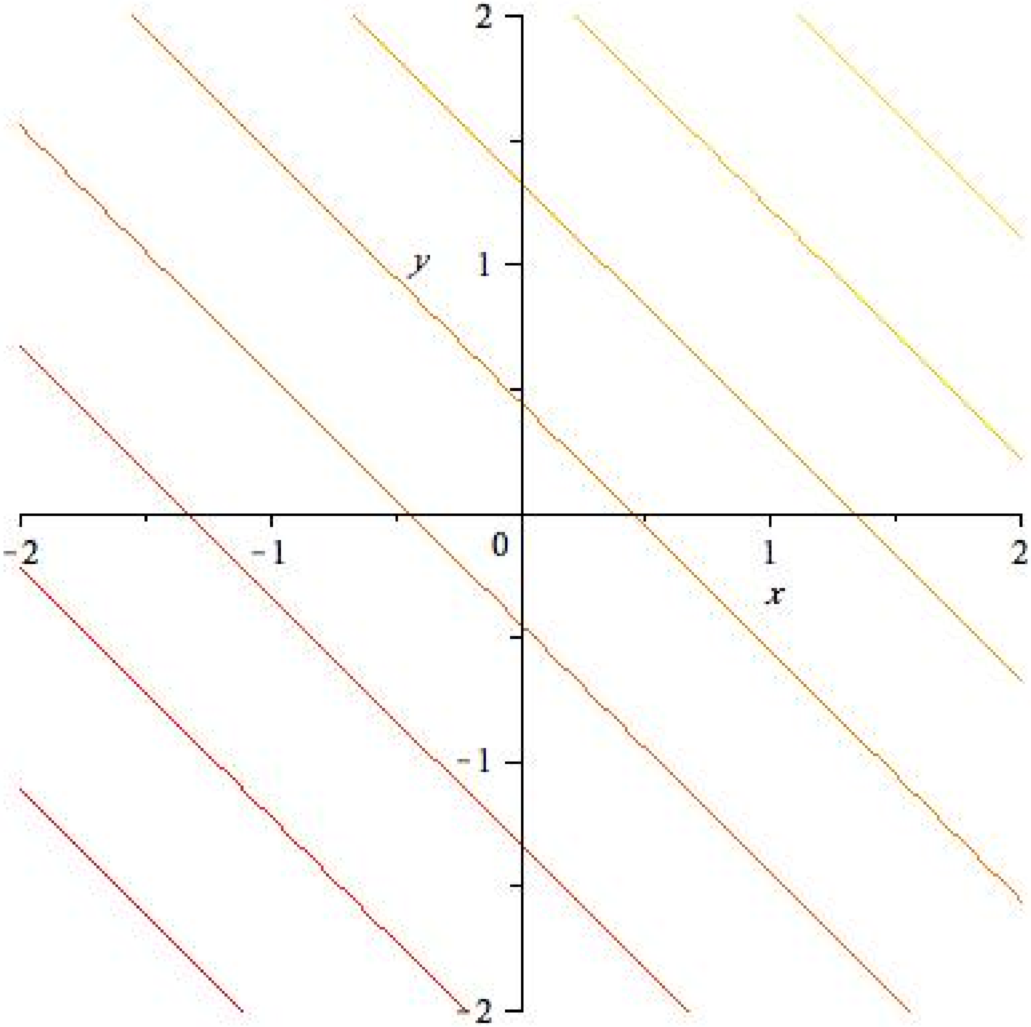

Figure 6: Level curves of f.

Other Practice Problems

1. Let $A = (2, 3, 2)$, $B = (3, 1, -1)$, and $C = (5, 2, 3)$.

   (a) Is the angle ABC acute, obtuse, or straight? Explain your answer.

   (b) Find the area of triangle ABC.

2. Determine whether the lines $\vec{r} = \langle 0,\ 1,\ 0\rangle + t\,\langle 1,\ 2,\ 2\rangle$ and $\vec{r} = \langle 2,\ 2,\ 6\rangle + t\,\langle 1,\ -1,\ 4\rangle$ intersect, and if so, find their point of intersection.

3. A moving particle has initial position $\vec{r}(0) = \hat{\imath} + \hat{\jmath} + \hat{k}$ and initial velocity $\vec{v}(0) = \hat{\imath} + \hat{k}$, and at time t has acceleration $\vec{a}(t) = -(\cos t)\hat{\imath} - (\sin t)\hat{\jmath} - (4t)\hat{k}$. Find the particle's position $\vec{r}(t)$ and velocity $\vec{v}(t)$ at time t.

4. Determine whether the limit exists, and if so, find its value.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3xy}{x^2 + y^2}$

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3xy^2}{x^2 + y^2}$

5. Consider the ellipsoid $2x^2 + 3y^2 + z^2 = 18$.

   (a) Find an equation of the tangent plane to the surface of the ellipsoid at the point $(-1, 2, -2)$.

   (b) Determine parametric equations describing the normal line at the same point.

6. The temperature at a point (x, y, z) is given by

$$T(x, y, z) = 3x^2 + y^2 - z.$$

   A moving particle has position at time t given by $\vec{r}(t) = \langle 2e^{-6t},\ 2e^{-2t},\ 2t\rangle$.

   (a) Show that at all times, the particle is moving in the direction in which temperature decreases fastest.

   (b) Find the rate of change of the particle's temperature with respect to time when $t = 0$.