Written Homework

for Math 11 students

September 8, 2012

In addition to WeBWorK problems, there will be some written homework in Math 11, generally one problem, and occasionally two problems, per class. There are at least two reasons for this.

In the past, some students have told us that WeBWorK homework did not give them enough practice for taking exams. WeBWorK does indeed give practice in solving problems, but it doesn't give practice in writing down your answer in a form that will get you full credit on an exam. (If the answer to a problem is 85, then typing "85" in WeBWorK gets you full credit. Writing "85" and nothing else on an exam gets you no credit, and possibly a suspicious question about where you got that answer if you haven't shown any work.) Doing written homework will give you practice in showing your work and explaining your answers in more than enough detail to get you full credit on an exam.

More important in the long run, if you plan to make any use of mathematics outside math courses, you will need to be able to explain your work in terms that non-mathematicians can understand. Some time ago, in a survey, employers of mathematics majors indicated that the one skill they valued most in the math majors they employed, and the one they most often found wanting, was the ability to communicate.

Because you have only one or two problems to write up, we expect you to do it right. Write in English and give clear and complete explanations of your work. Write as if you were explaining a problem to another student, who understands calculus but has not seen this problem, and will have to understand your written explanation without having you there in person to answer questions. Try reading your solution aloud, and see how it sounds.

Here are some details about written homework:

1. Written homework assigned each class day is due by 9:00 AM on the day of the next class.

2. Written homework will be submitted and returned using the homework boxes on the first floor of Kemeny Hall. We are required to ask you to sign a waiver for this, because it is possible that somebody else can look at the homework in the boxes and see what grade you are assigned. If you do not want to sign this waiver, you can pick up your homework at your instructor's office hours.

3. Written homework is graded based on both the correctness of the mathematics and the correctness, completeness, and clarity of the explanation. A correct answer with an explanation that is hard to read, or incomplete, or illegible, does not get full credit. Graders are not expected to translate illegible writing or unclear English in an attempt to give you credit for whatever correct mathematics you might have done.

4. Written homework is graded on a scale of 0 to 5. A grade of 1 indicates that you have apparently made some attempt at the problem. A grade of 2 indicates that your solution manages to communicate some progress on the problem. A grade of 3 indicates either a somewhat flawed explanation of a partial solution, or an excellent explanation of the problem and attempted solution (even though the mathematics might be completely wrong), or a correct mathematical solution with a very inadequate explanation. A grade of 4 indicates either an excellent explanation of a solution that is partially correct, or a somewhat flawed explanation of a complete and correct solution. A grade of 5 indicates an excellent explanation of a complete and correct solution.

5. The honor principle applies to written homework in the following way: You may work together on homework, but you must write up your answers yourself without reference to anyone else. It is not acceptable, for example, to work together with a group on a problem and then to copy down the group solution. It is perfectly acceptable (and encouraged!) to work together with a group on a problem, or to ask for help from a tutor or a friend, as long as you then write up the answer by yourself, using your own words.

As an example, here are two different solutions to Problem 11 of Section 13.1. Notice the following important points:

1. Each solution begins by restating the problem. It is not sufficient to say "Section 13.1, Problem 11," and then launch directly into solving the problem; your solution should be self-contained.

2. The solutions are written in English using complete sentences. Mathematics is communicated in English. On exams, when you have limited time, we will not expect as much[1]; as long as your solution is clear, you show your steps and explain them where necessary, and we can follow your work easily, you will get credit for the work. In the real world, employers and colleagues of mathematicians and other quantitatively-inclined people expect explanations they can read or listen to.

3. The solutions use equations, formulas and pictures. Often a picture or an equation is the clearest way of communicating.

4. The solutions *explain* the equations, formulas and pictures. For example, the first solution explains that the first equation given is the general equation for a sphere, and then explains how to get from the first equation to the second. A string of equations without explanations, such as

$$(x-h)^2 + (y-k)^2 + (z-\ell)^2 = r^2$$

$$\boxed{(x-1)^2 + (y+4)^2 + (z-3)^2 = 25}$$

$$(x-1)^2 + 16 + (z-3)^2 = 25$$

$$(x-1)^2 + (z-3)^2 = 9$$

$$\boxed{\text{The circle in the } xz\text{-plane with center } (1,0,3) \text{ and radius } 3,}$$

   is not a write-up.

5. How your solution is laid out on the paper matters. Centering equations on their own lines, and skipping lines between parts of a solution, can make your solution much more readable. Neatness counts.

6. It is fine to use formulas from the text or from class, such as the equation for a sphere or the distance formula.

[1]No, I don't mean that on exams you can explain your solutions in Latin.

7. There is generally more than one correct solution. The first solution below approaches the second part of the problem algebraically (working with equations and formulas) while the second solution approaches it geometrically. Unless the problem specifies a particular approach or technique, you can solve it in any way that is mathematically valid.

8. Good mathematical style does not consist of complex and varied grammatical constructions and word choice; often it consists of simple, repetitive prose. Clarity and precision are what matter.

Section 13.1, Problem 11

Find an equation of the sphere with center (1, -4, 3) and radius 5. What is the intersection of this sphere with the xz-plane?

Solution: We use the general equation of a sphere with center (h, k, ℓ) and radius r,

$$(x-h)^2 + (y-k)^2 + (z-\ell)^2 = r^2,$$

and substitute in the given center and radius to get

$$\boxed{(x-1)^2 + (y+4)^2 + (z-3)^2 = 25.}$$

The equation of the xz-plane is $y = 0$, so the intersection of the sphere and the plane will be all points that satisfy both equations. Substituting $y = 0$ in the equation for the sphere, we get

$$(x-1)^2 + 16 + (z-3)^2 = 25,$$

or, subtracting 16 from each side,

$$(x-1)^2 + (z-3)^2 = 9.$$

Therefore, the intersection is all points in the xz-plane satisfying this equation.

Viewed as an equation in two dimensions, this is the equation of a circle having center $(x, z) = (1, 3)$ and radius 3. Therefore, the intersection is:

$$\boxed{\text{The circle in the } xz\text{-plane with center } (1,0,3) \text{ and radius 3.}}$$

13.1 # 11

Problem: Find an equation for the sphere with center (1, −4, 3) and radius 5.

Solution: We want all points whose distance from the point (1, −4, 3) is 5. That's the same thing as saying the square of the distance is 25. The distance formula in three dimensions (page 803) gives the equation:

$$(x-1)^2 + (y+4)^2 + (z-3)^2 = 25.$$

Problem: Find the intersection of this sphere with the xz-plane.

Solution: The intersection is a circle.

The line through the center of our sphere, (1, −4, 3), and perpendicular to the xz-plane, is the line $x = 1$, $z = 3$, which intersects the xz-plane (the plane $y = 0$) in the point (1, 0, 3). This is the center of the circle.

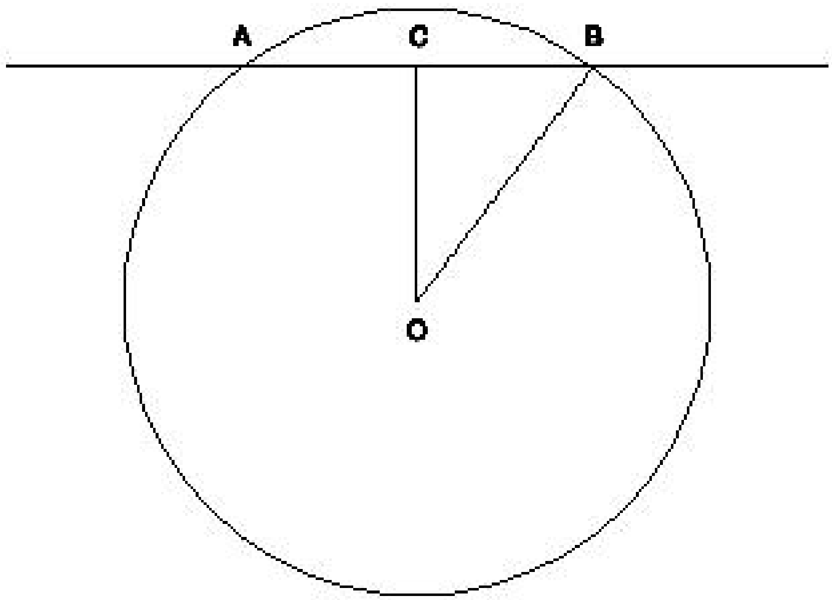
This picture (on the next page) is a cross-section; the large circle is a cross-section of the sphere, the horizontal line is the xz-plane, and the vertical line is perpendicular to the xz-plane. The center of the sphere is O, the center of the circle is C, and the line AB is a diameter of the circle. The distance from C to B is the radius of the circle.

Triangle OCB is a right triangle. The distance from $O(1, -4, 3)$ to $C(1, 0, 3)$, given by the distance formula in 3 dimensions, is

$$\sqrt{(1-1)^2 + (-4-0)^2 + (3-3)^2} = 4.$$

The distance from O to B is the radius of the sphere, 5. By the Pythagorean Theorem, the distance from C to B is 3.

The intersection of the sphere with the xz-plane is the circle in the xz-plane with center (1, 0, 3) and radius 3.


A
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O