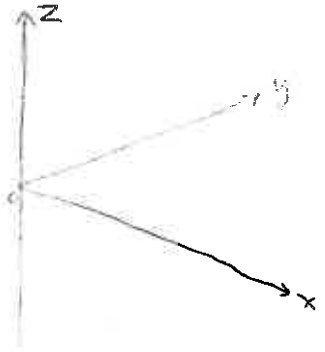
MATH 32 WORKSHEET: Vector fcns. 01/29/10 Barrett

A) Sketch in 3d the curve given by $\vec{r}(t) = \langle t, t, \sin t \rangle$



[What plane does it lie in?]

B) Compute $\vec{r}\,'(t) = \langle \quad , \quad , \quad \rangle$

What is the speed as a function of t?

Use this to write the unit tangent vector $\vec{T} = \langle$

C) Can you write a different vector fcn. that generates the same space curve as $\vec{r}(t)$?
If so, give an example:

D) Find the parametric eqn for tangent line to $\vec{r}(t)$ at $t = \pi$:
[Hint: find $\vec{r}_0$ and $\vec{v}$ for line]

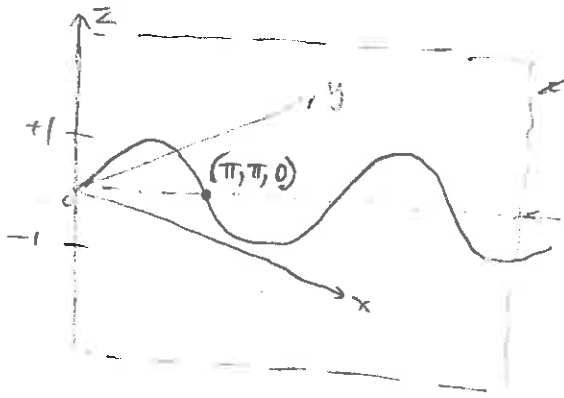
if time...

E) If $\vec{a}(t)$ & $\vec{b}(t)$ are vector fcns, write out $\vec{a}\cdot\vec{b}$ in coordinates,
($\vec{a} = \langle a_1, a_2, a_3 \rangle$, $\vec{b} = \langle b_1, b_2, b_3 \rangle$)
then take $\frac{d}{dt}$ to get formula for $\frac{d}{dt}(\vec{a}\cdot\vec{b})$ in terms of $\vec{a}, \vec{b}, \vec{a}\,', \vec{b}\,'$

MATH 21 WORKSHEET: Vector fcns. 11/24/10 Barnett

SOLUTIONS

A) Sketch in 3d the curve given by $\vec{r}(t) = \langle t, t, \sin t \rangle$



curve lies in plane $x = y$

line $x=y, z=0$

[Which plane does it lie in?]

B) Compute $\vec{r}\,'(t) = \langle 1, 1, \cos t \rangle$

What is the speed as a function of t? $|\vec{r}\,'(t)| = \sqrt{1^2+1^2+\cos^2 t} = \sqrt{2+\cos^2 t}$

Use this to write the unit tangent vector $\vec{T} = \left\langle \frac{1}{\sqrt{2+\cos^2 t}}, \frac{1}{\sqrt{2+\cos^2 t}}, \frac{\cos t}{\sqrt{2+\cos^2 t}} \right\rangle$

C) Can you write a different vector fcn. that generates the same space curve as $\vec{r}(t)$?

If so, give an example: $\langle 2t, 2t, \sin 2t \rangle$

$\langle t+1, t+1, \sin(t+1) \rangle$

$\langle t^2, t^2, \sin(t^2) \rangle$ ← note doesn't produce negative parts of curve

D) Find the parametric eqn for tangent line to $\vec{r}(t)$ at $t = \pi$:

[Hint: find $\vec{r}_0$ and $\vec{v}$ for line]

$\vec{r}_0 = \vec{r}(\pi) = (\pi, \pi, 0)$

velocity $\vec{v} = \vec{r}\,'(\pi) = (1, 1, -1)$

Put together $\vec{r} = \vec{r}_0 + \vec{v}t$ ie $\begin{cases} x = \pi + t \\ y = \pi + t \\ z = -t \end{cases}$ parametric eqns. for line.

if time... a new parameter t! (careful: t not same meaning as in $\vec{r}\,'(t)$)

E) If $\vec{a}(t)$ & $\vec{b}(t)$ are vector fcns, write out $\vec{a}\cdot\vec{b}$ in coordinates.

(a_1, a_2, a_3) (b_1, b_2, b_3) $a_1 b_1 + a_2$

then take $\frac{d}{dt}$ to get formula for $\frac{d}{dt}(\vec{a}\cdot\vec{b})$ in terms of $\vec{a}, \vec{b}, \vec{a}\,', \vec{b}\,'$

$(a_1 b_1 + a_2 b_2 + a_3 b_3)' = a_1' b_1 + a_1 b_1' + a_2' b_2 + a_2 b_2' + a_3' b_3 + a_3 b_3' = \underline{\vec{a}\,'\cdot\vec{b} + \vec{a}\cdot\vec{b}\,'}$