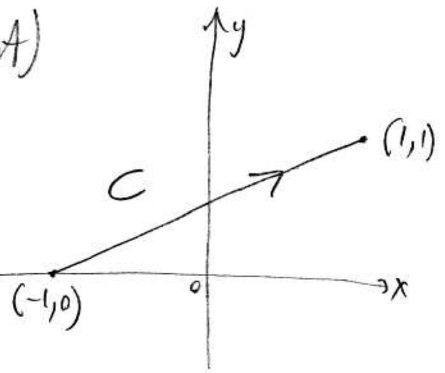
Barnett 11/5/10

MATH 11 WORKSHEET: Scalar line integrals

A)



C = straight line with sense shown.

i) Parametrize C:

$x(t) = \cdots$

$y(t) = \cdots$

ii) Write length element ds in terms of dt: [Hint: $|\vec{r}\,'(t)|$]

iii) Write $f(x,y) = xy^2$ in terms of t on this "curve":

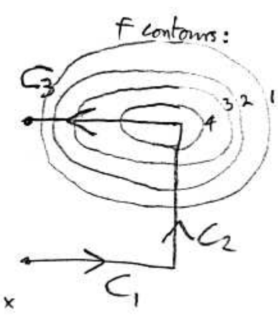
$$f(\vec{r}(t)) = \ldots$$

iv) Hence get $\int_C f\, ds = \ldots$

v) Now find $\int_C (x+y)\,dx = \int_0^1 (-1+2t+t)\,2dt = 2\left(\frac{3}{2}-1\right) = 1$

vi) If C were taken in reverse, how would answers iv) & v) change?

B)



$C = C_1 + C_2 + C_3$

Decide $+$, $-$, or 0:

	$\int f\,dx$	$\int f\,dy$	$\int f\,ds$
C_1			0
C_2			
C_3			
C			
C traversed backwards → $-C$			

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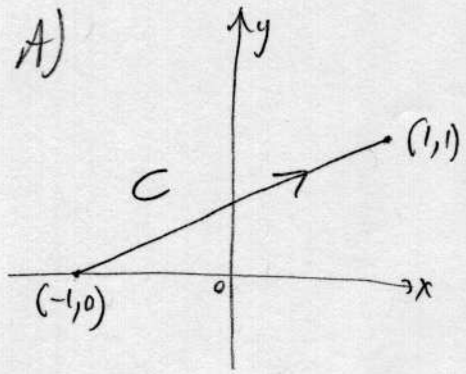
A)



C = straight line with sense shown.

i) Parametrize C:

$x(t) = -1+2t \qquad x' = 2$

$y(t) = t \qquad y' = 1$

ii) Write length element ds in terms of dt: $|\vec{r}'| = \sqrt{x'^2 + y'^2}$ [Hint: $|\vec{r}'(t)|$

$\Rightarrow ds = \sqrt{5}\,dt$ $\qquad = \sqrt{5}$ indep. of t.

iii) Write $f(x,y) = xy^2$ in terms of t on this "curve":

$$f(\vec{r}(t)) = \dots (-1+2t)t^2$$

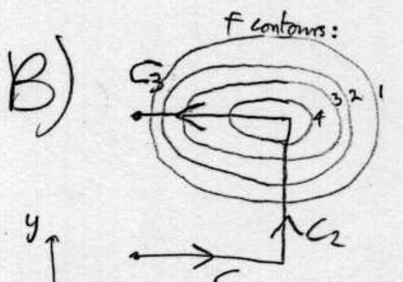
iv) Hence get $\int_C f\,ds = \int_0^1 (2t^3 - t^2)\sqrt{5}\,dt = \sqrt{5}\left(\frac{2}{4} - \frac{1}{3}\right) = \frac{\sqrt{5}}{6}$

v) Now find $\int_C (x+y)\underbrace{dx}_{x'dt = 2dt} = \int_0^1 (-1+2t+t)\,2dt = 2\left(\frac{3}{2} - 1\right) = 1$

vi) If C were taken in <u>reverse</u>, how would answers iv) & v) change?

iv): stays same (like arc length) $\qquad$ v): swapped sign (like $\int_a^b dt$)

B)



$C = C_1 + C_2 + C_3$

Decide +, −, or 0:

	$\int f\,dx$	$\int f\,dy$	$\int f\,ds$
C_1	0	0	0
C_2	0	+	+
C_3	−	0	+
C	−	+	+
C traversed backwards → $-C$	+	−	+