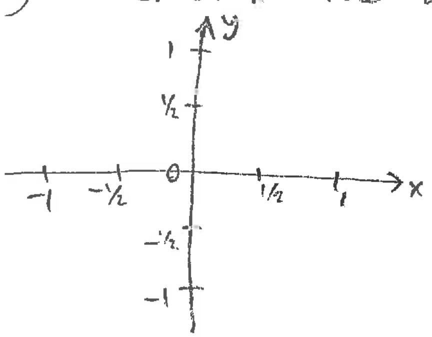
10/18/10
Barnett

MATH 11 WORKSHEET: Lagrange multipliers

A) Find the min & max of $f(x,y) = 3 + x + 4y$
restricted to the curve $x^2 + 4y^2 = 1$

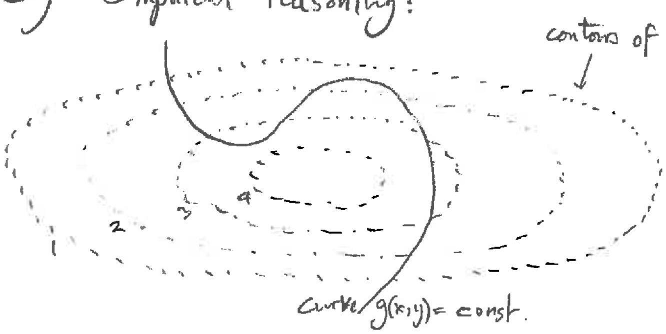
abs max =
abs min =

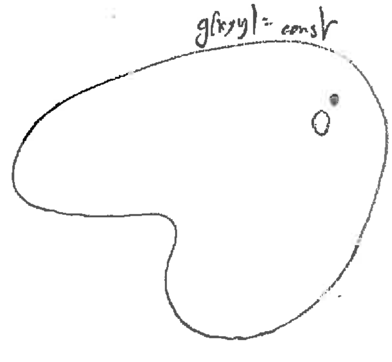
B) Sketch the domain $x^2 + 4y^2 \leq 1$



Use the above to find the absolute maximum of $3 + x + 4y$ in this domain. [Hint: try find interior critical pts]
Add this solution, and f contours, to sketch.

C) Graphical reasoning:





f = distance to origin O

Label all local & absolute extrema

Find local extrema of f restricted to curve
Draw $\vec{\nabla} f$ at these points, and $\vec{\nabla} g$ ← you have one decision to make!

Add contours of f, & $\vec{\nabla} f$ at extrema.

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SOLUTIONS

A) Find the min & max of $f(x,y) = 3 + x + 4y$ $\quad \nabla f = (1, 4)$

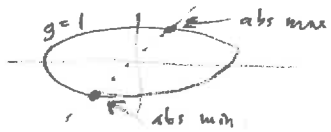
restricted to the curve $\underbrace{x^2 + 4y^2}_{g(x,y)} = 1$

3 unknowns, 3 eqns: $\quad \vec{\nabla} g = (2x, 8y)$

① $1 = 2\lambda x$ $\quad \lambda = \frac{1}{2x}$ sub in

② $4 = 8\lambda y$ $\quad$ so $4 = 8\frac{1}{2x}y$ ie $4x = 4y$, $x = y$.

③ $x^2 + 4y^2 = 1$ $\xrightarrow{\text{sub } x=y}$ $x^2 + 4x^2 = 1$ $\quad x = \pm\frac{1}{\sqrt{5}}$

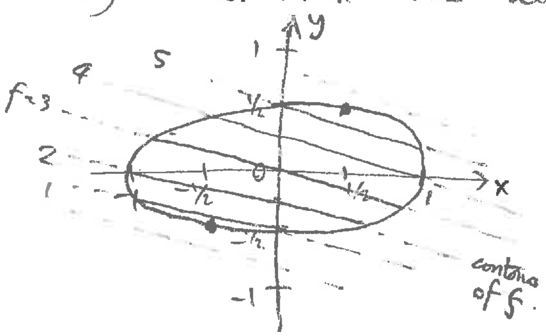


so $f\left(\frac{1}{\sqrt5}, \frac{1}{\sqrt5}\right) = 3 + \frac{1}{\sqrt5} + \frac{4}{\sqrt5} = 3 + \sqrt5$

$f\left(-\frac{1}{\sqrt5}, -\frac{1}{\sqrt5}\right) = 3 - \frac{1}{\sqrt5} - \frac{4}{\sqrt5} = 3 - \sqrt5$

$\boxed{\begin{array}{l}\text{abs max} = 3+\sqrt5 \\ \text{abs min} = 3-\sqrt5\end{array}}$

B) Sketch the domain $x^2 + 4y^2 \le 1$ $\quad$ ellipse, interior & boundary thereof.

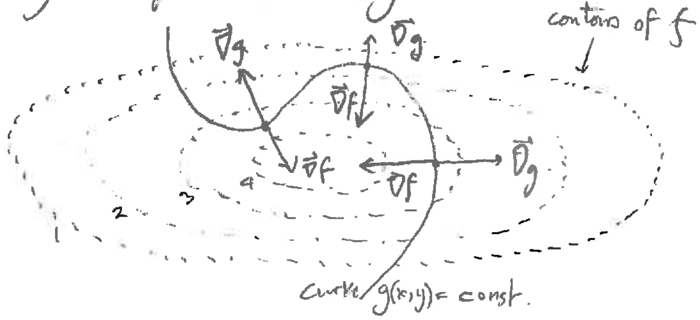


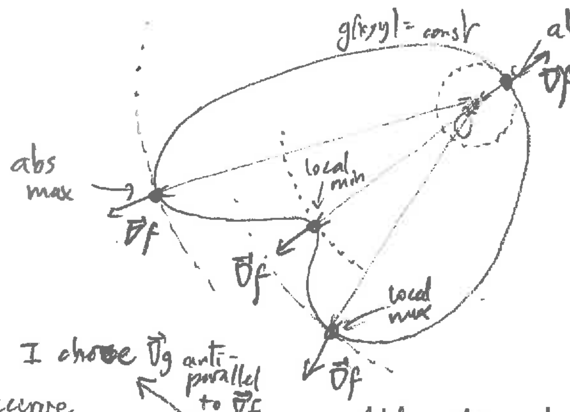
Use the above to find the absolute maximum of $3 + x + 4y$ in this domain. [Hint: try find interior critical pts] Add this solution, and f contours, to sketch.

no interior crit. pts since $\vec{\nabla} f = (1,4)$ is never $= (0,0)$!

$\Rightarrow$ Only extrema are on bdry, and we already got them in A). $\Rightarrow$ abs max & min same as A)

C) Graphical reasoning:





Label all local & absolute extrema

Find local extrema of f restricted to curve

Draw $\vec{\nabla} f$ at these points, and $\vec{\nabla} g$ ← you have one decision to make!

I chose $\vec{\nabla} g$ anti-parallel to $\vec{\nabla} f$.

Add contours of f, & $\vec{\nabla} f$ at extrema.