Your name:

Instructor (please circle): Barnett Van Erp

Math 11 Fall 2010: written part of HW4 (due Wed Oct 20)

Please show your work. No credit is given for solutions without justification.

(1) [8 points] A spaceship is traveling in space when its engines fail. Without the engines the ship starts to drift towards a star. The temperature in space changes according to the function

$$T(x, y, z) = \frac{4,900}{2x^2 + y^2 + z^2}.$$

The engineers are able to restart the engine when they are at position $(1, 2, 1)$.

(a) [3 points] Calculate the direction in which they should proceed to **cool** the spaceship most rapidly. Express your answer in the form of a **unit vector**.

(b) [2 points] At what rate (degrees per unit distance) will it cool if they go in that direction?

(c) [3 points] At the present speed, the ship should not cool of at a rate that exceeds 300 degrees per unit distance. This means that the ship can not travel in the direction you found in item (a). Find the angle θ between the direction of **maximum decrease** of the temperature T and the direction in which the temperature decreases at exactly 300 degrees per unit distance. (Note: the position of the ship is still at coordinates $(1, 2, 1)$.)

(2) [8 points] A *sphere* with center $(3, 5, 2)$ and radius $\sqrt{21}$ can be represented as the level surface $g(x, y, z) = 21$ of the function of three variables

$$g(x, y, z) = (x - 3)^2 + (y - 5)^2 + (z - 2)^2.$$

The graph $z = f(x, y)$ of the function of two variables

$$f(x, y) = x^2 + 2y^2$$

is a surface called an *elliptic paraboloid*.

These two surfaces intersect at the point $(x, y, z) = (1, 1, 3)$. Determine whether the two surfaces are tangent at this point. In other words: determine if the tangent plane to the sphere at point $(1, 1, 3)$ is the *same plane* as the tangent plane to the elliptic paraboloid at this point. Explain your answer.

(3) [10 points]

(a) [3 points] Find all critical points of the function $f(x, y) = x^2 + y^2 + x^2y + 4$.

(b) [3 points] For each critical point, determine whether it is a local maximum, a local minimum, or a saddle point.

(c) [4 points] Find the **absolute** maximum of the function $f(x, y)$ on the set

$$D = \{(x, y) \mid 0 \leq x \leq 2, -2 \leq y \leq 0.\}$$

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