Your name:

Instructor (please circle): Barnett Van Erp

Math 11 Fall 2010: written part of HW3 (due Wed Oct 13)

Please show your work. No credit is given for solutions without justification.

(1) [10 points] For each of the limits in items (a) and (b) below, either find the limit and prove that it is what you claim it is, or else prove that it does not exist.

(a) [3 points]

$$\lim_{(x,y)\to(0,0)} \frac{x^5 + x^2y^2}{x^4 + y^4}$$

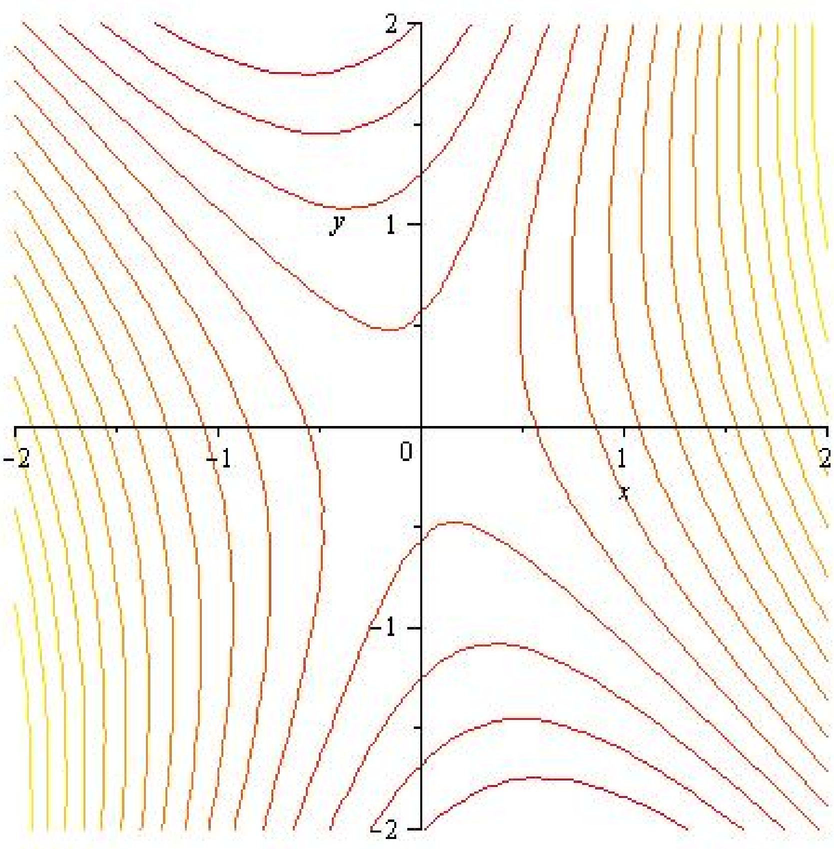
(b) [3 points]

$$\lim_{(x,y)\to(0,0)} \frac{(x+y)^3}{x^2 + y^2}$$

(c) [4 points] Find all points where the function $f(x, y)$ defined below is continuous. Justify your answer.

$$f(x,y) = \begin{cases} \cos(xy) & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

(2) [9 points] Below is part of the contour map for a two-variable function $f(x, y)$. The values of $f(x, y)$ for which the level curves are depicted are at chosen at constant intervals. The red curves represent lower function values, while the yellow curves represent higher function values.



(a) [2 points] What seem to be the values of $f_x(0,0)$ and $f_y(0,0)$? Justify your answer.

(b) [2 points] What is the sign of the second derivative $f_{xx}(1,1)$? Justify your answer.

(c) [2 points] What is the sign of the second derivative $f_{yy}(0,-2)$? Justify your answer.

(d) [3 points] What is the sign of the mixed derivative $f_{xy}(0,1)$? Justify your answer.

(3) [8 points] For items (a), (b), (c) below we let $f(x, y) = \sqrt{49 - x^2 - y^2}$. The graph of this function is a hemisphere.

(a) [3 points] Find an equation for the tangent plane to the graph of the function $f(x, y)$ at the point $(6, 3, 2)$.

(b) [3 points] Calculate the angle between the xy plane and the tangent plane that you found in part (a).

(c) [2 points] Is the function $f(x, y)$ differentiable at $(x, y) = (6, 3)$? Justify your answer.

(d) [BONUS, 2 points] Do you think $f(x, y)$ is differentiable at $(x, y) = (7, 0)$? Explain your answer.