Math 11 Fall 2010: written part of HW1 (due Wed Sept 29)

Please give your working. The space indicates the relative weight of each question sub-part.

1. [10 points]

   (a) A force of size 10 pulls at an angle of $60°$ to the x-axis on an object that moves from location $(5, 1)$ to location $(1, 1)$. How much work is done by the force?

   (b) Find a *unit* vector orthogonal to the vectors $\mathbf{a} = (1, 2, 3)$ and $\mathbf{b} = (1, 0, -1)$.

   [BONUS: How many such unit vectors exist?]

(c) Find the volume of the parallelepiped formed by the vectors **a** and **b** above, and a third vector **c** $= (2, 1, 0)$

2. [7 points] A planar wall is described by its normal vector **n**. After a particle with incoming velocity vector **v** bounces elastically off this wall, its outgoing velocity vector is $\mathbf{u} = \mathbf{v} - 2\text{proj}_\mathbf{n}\mathbf{v}$.

(a) What outgoing velocity does a particle with incoming velocity $(1, 2, 3)$ have after bouncing off a wall whose normal is $(1, 0, -1)$?

(b) Show that for a general **v** and **n** the speed (magnitude of velocity vector) is unchanged by the bounce. [Hint: Use the formula for $\text{proj}_\mathbf{n}\mathbf{v}$ and then compute $\mathbf{u} \cdot \mathbf{u}$. Then show that this is equal to $\mathbf{v} \cdot \mathbf{v}$.]

3. [10 points]

(a) Find the equation for a sphere with center $(4, -2, 1)$ which also passes through the point $(8, 1, 1)$

(b) Find the radius and center of the circle given by the intersection of this sphere with the yz coordinate plane.

(c) The vector equation $(\mathbf{r} - \mathbf{a}) \cdot (\mathbf{r} - \mathbf{b}) = 0$ describes a sphere. If $\mathbf{a} = \langle 2, 0, 5 \rangle$ and $\mathbf{b} = \langle 6, 4, 3 \rangle$, find the center and radius of the sphere. [Hint: you may want to use Cartesian coordinates $\mathbf{r} = \langle x, y, z \rangle$]