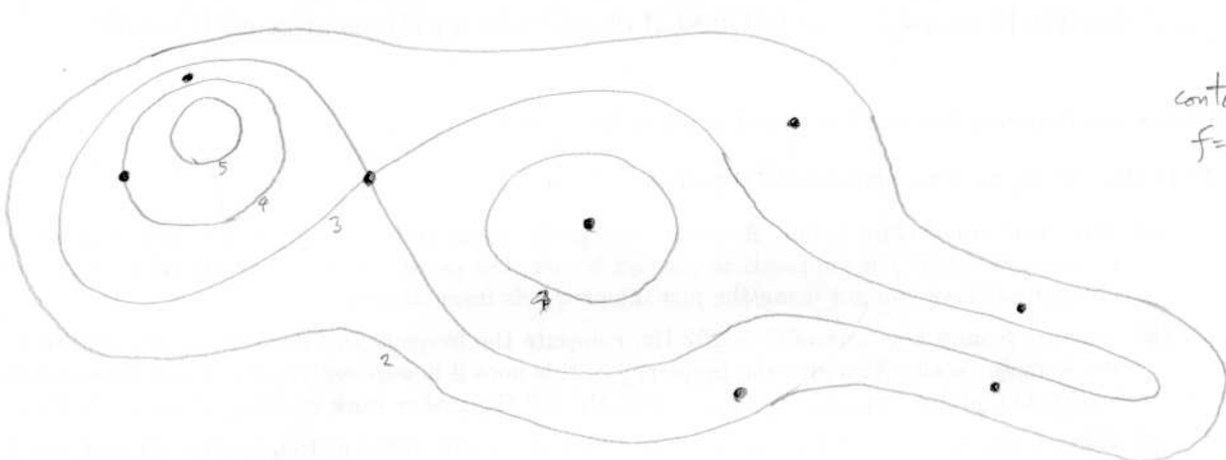
MATH 11 WORKSHEET : Gradient vector in 2d

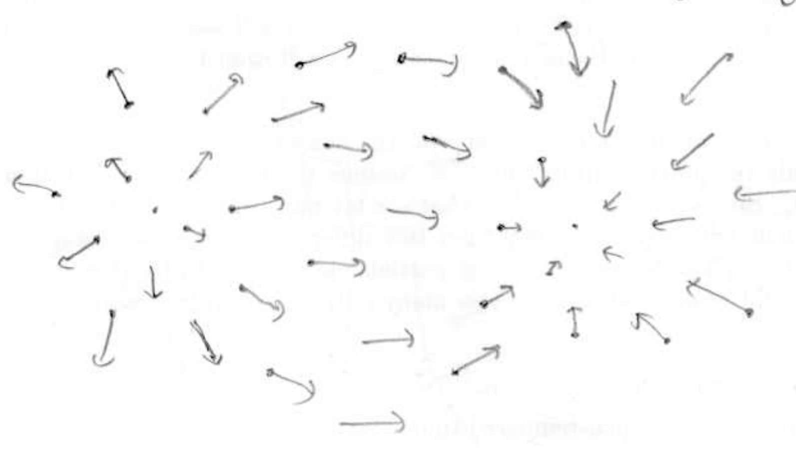
Barnett
10/13/10

A) Draw vectors at the points shown (as dots), showing $\vec{\nabla} f$ at each. If $\vec{\nabla} f$ has zero length, explain why.



contours show $f = 2, 3, 4, \cdots$

B) Add contours of f for the function whose $\vec{\nabla} f$ vectors are shown at the given points (f is smooth so you may "fill in" missing areas sensibly):



- where is the highest pt? (hill)
- where is the lowest (bow

C) Compute $\vec{\nabla} f$ for $f(x,y) = \sqrt{x^2+y^2}$

What is $\vec{\nabla} f\,(4,3)$?

What is $|\vec{\nabla} f|$ for general x,y? (simplify).

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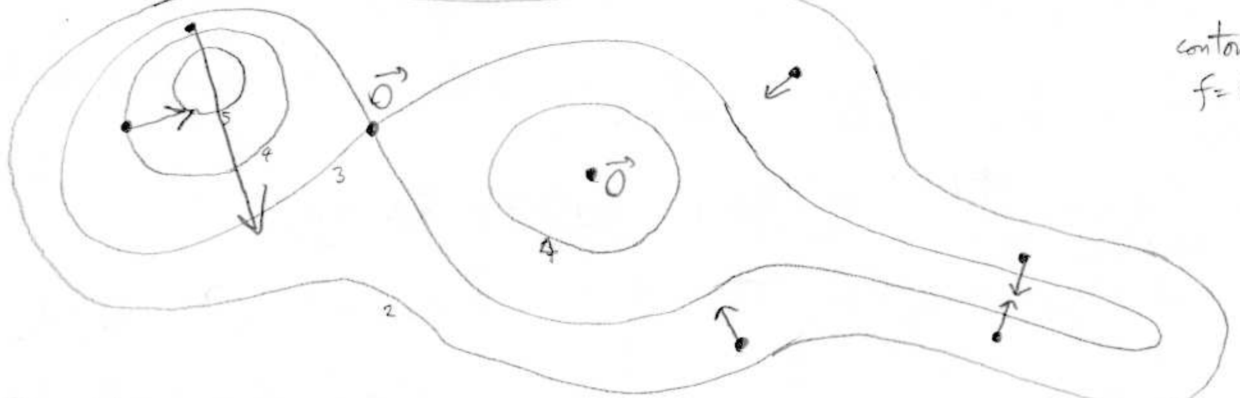
SOLUTIONS

A) Draw vectors at the points shown (as dots), showing $\vec{\nabla} f$ at each. If $\vec{\nabla} f$ has zero length, explain why.

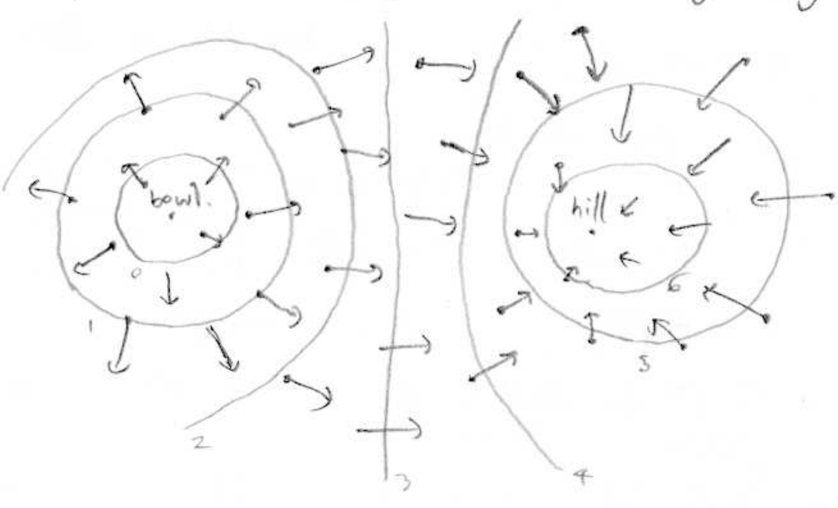
note $|\vec{\nabla} f|$ large where steep : $|\vec{\nabla} f|$ = steepness $\left(= \frac{1}{\text{spacing of contour lines}}\right)$

contours show $f = 2, 3, 4, \cdots$



$\vec{\nabla} f = \vec{0}$ at saddle since $\vec{\nabla} f$ must be $\perp$ to 2 nonparallel contour lines at that pt.

B) Add contours of f for the function whose $\vec{\nabla} f$ vectors are shown at the given points (f is smooth so you may "fill in" missing areas sensibly):



- where is the highest pt? (hill)
- where is the lowest (bow

C) Compute $\vec{\nabla} f$ for $f(x,y) = \sqrt{x^2+y^2}$

$f_x = 2x \cdot +\frac{1}{2}(x^2+y^2)^{-1/2} = \frac{+x}{\sqrt{x^2+y^2}}$, $f_y = \frac{+y}{\sqrt{x^2+}}$

so $\vec{\nabla} f = \left(\frac{+x}{\sqrt{x^2+y^2}}, \frac{+y}{\sqrt{x^2+y^2}}\right)$

What is $\vec{\nabla} f\,(\underset{x}{4},\underset{y}{3})$? $\left(\frac{+4}{5}, \frac{+3}{5}\right)$

$z = \sqrt{x^2+y^2}$ is cone: constant steepness = 1!

What is $|\vec{\nabla} f|$ for general x, y? (simplify). $|\vec{\nabla} f| = \sqrt{\frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2}} = 1$

for all (x,y) except $(0,0)$.